Temporal Vertex Cover with a Sliding Time Window

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These results have been presented in **ICALP 2018**

Joint work with:

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AlgoUK, University of Liverpool
September 2018

Static and Temporal Graphs

Modern networks are highly dynamic:

- Social networks: friendships are added/removed, individuals leave, new ones enter
- Transportation networks: transportation units change with time their position in the network
- Physical systems: e.g. systems of interacting particles

The common characteristic in all these applications:

- the graph topology is subject to discrete changes over time
- ⇒ the notion of vertex adjacency must be appropriately re-defined (by introducing the time dimension in the graph definition)

Various graph concepts (e.g. reachability, connectivity):

- crucially depend on the exact temporal ordering of the edges

Temporal graphs

Formally:

Definition (Temporal Graph)

A temporal graph is a pair (G, λ) where:

- $G = (V, E)$ is an underlying (di)graph and
- $\lambda : E \to 2^{\mathbb{N}}$ is a discrete time-labeling function.

Temporal graphs

Formally:
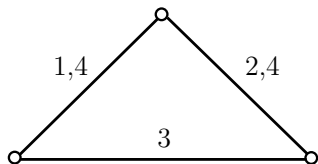

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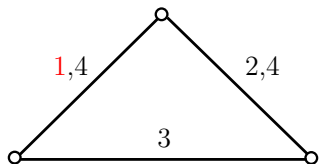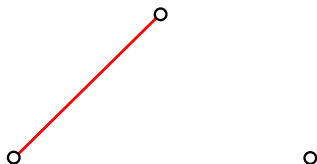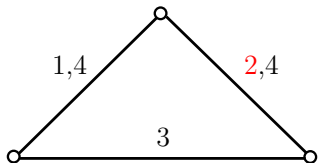
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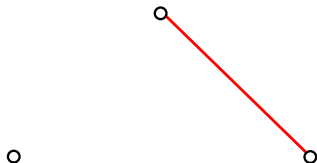
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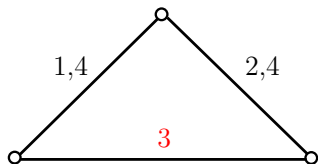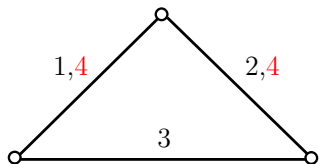
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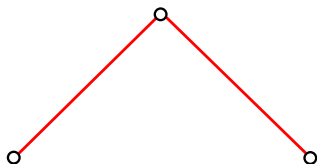Formally:

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Alternatively, we can view it as a sequence of static graphs, the snapshots:
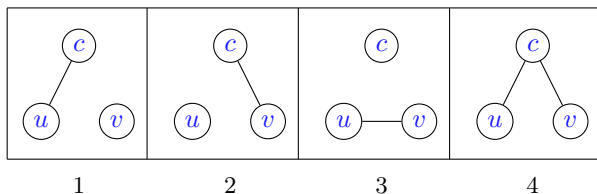
Example: static vs temporal graphs
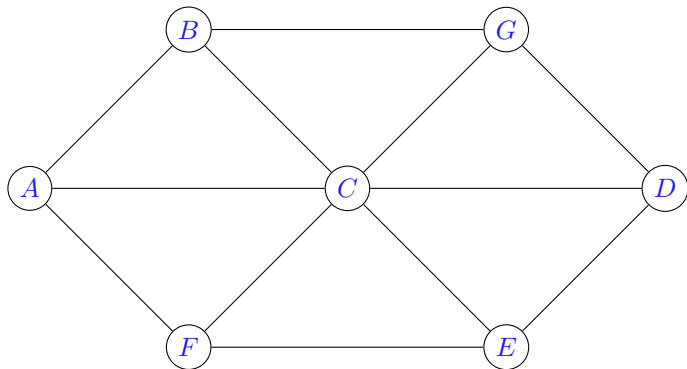
From paths to temporal paths:

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Overview
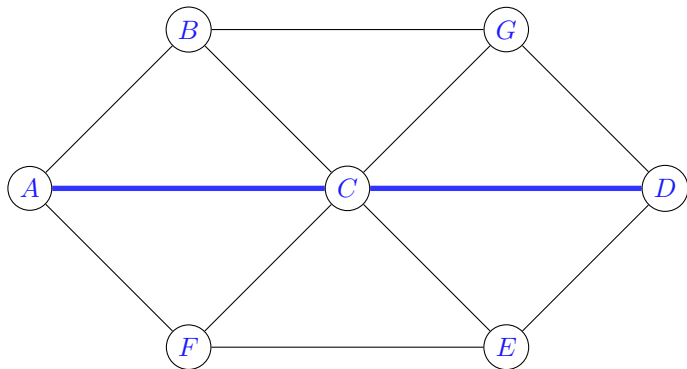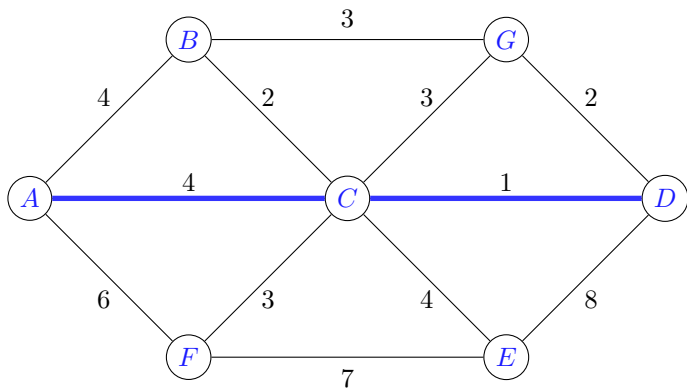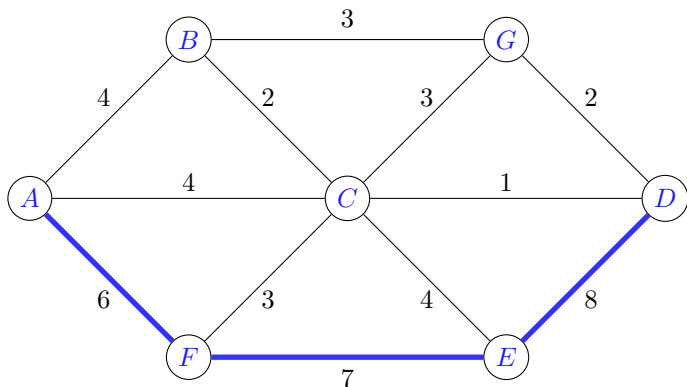
- Basic definitions

- Temporal vertex cover

- Temporal vertex cover with a sliding time window

- Open problems

Basic definitions I

To specify a temporal graph class, we can:

- either restrict the underlying graph G,
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Definition (Temporal Graph Classes)

For a class $\mathcal{X}$ of static graphs we say that a temporal graph (G, λ) is

- $\mathcal{X}$ temporal, if $G \in \mathcal{X}$;
- always $\mathcal{X}$ temporal, if $G_i \in \mathcal{X}$ for every $i \in [T] = \{1, 2, \ldots, T\}$.

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Definition (Temporal Vertex Subset)

A pair $(u, t) \in V \times [T]$ is called the appearance of vertex u at time t.
A temporal vertex subset of (G, λ) is a set $\mathcal{S} \subseteq V \times [T]$ of vertex appearances in (G, λ).

Basic definitions II

Definition (Edge is Temporally Covered)

A vertex appearance (w, t) temporally covers an edge e if:

(i) w covers e, i.e. $w \in e$, and

(ii) $t \in \lambda(e)$, i.e. the edge e is active during the time slot t.

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Example:

$G_1 \qquad G_2 \qquad G_3 \qquad G_4 \qquad G_5 \qquad G_6 \qquad G_7 \qquad G_8$
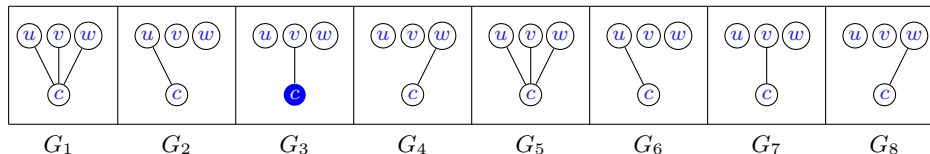
– $(c, 3)$ temporally covers edge cv, but

– $(c, 3)$ temporally covers neither cu, nor cw.

Basic definitions: Temporal Vertex Cover

Definition (Temporal Vertex Cover)

A temporal vertex cover of (G, λ) is a temporal vertex subset $\mathcal{S}$ of (G, λ) such that every edge $e \in E(G)$ is temporally covered by at least one vertex appearance in $\mathcal{S}$.

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Example

– $\{(c, 2), (c, 3), (c, 8)\}$ is a Temporal Vertex Cover

Basic definitions: Temporal Vertex Cover

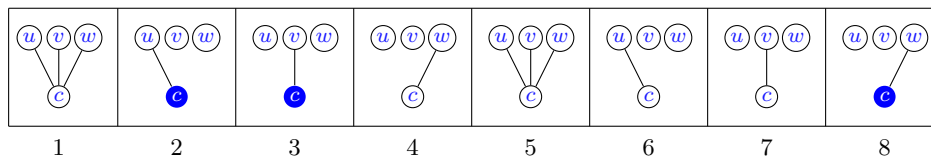## Definition (Temporal Vertex Cover)

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Example

- $\{(c, 2), (c, 3), (c, 8)\}$ is a Temporal Vertex Cover
- $\{(c, 5)\}$ is a minimum Temporal Vertex Cover

Basic definitions: Temporal Vertex Cover

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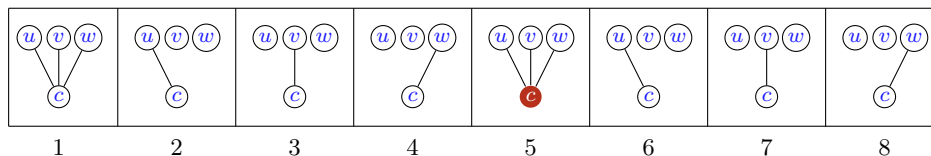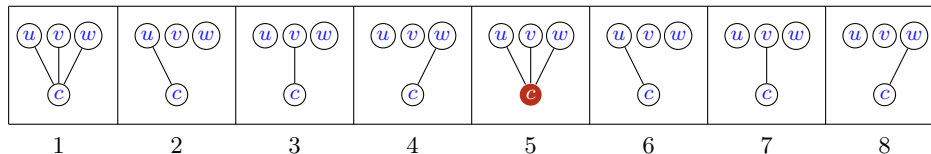
Example

TEMPORAL VERTEX COVER (TVC)

Input: A temporal graph (G, λ).
Output: A temporal vertex cover $\mathcal{S}$ of (G, λ) with the minimum $|\mathcal{S}|$.

Definition (Time Windows)

1. For every time slot $t \in [1, T - \Delta + 1]$:
   the time window $W_t = [t, t + \Delta - 1]$ is the sequence of the
   Δ consecutive time slots $t, t + 1, \ldots, t + \Delta - 1$.

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3. $\mathcal{S}[W_t] = \{(w, t) \in \mathcal{S} : t \in W_t\}$ is the restriction of the temporal vertex subset $\mathcal{S}$ to the window W_t.

Definition (Sliding Δ-Window Temporal Vertex Cover)

A sliding Δ-window temporal vertex cover of (G, λ) is a temporal vertex subset $\mathcal{S}$ of (G, λ) such that:

- for every time window W_t and for every edge $e \in E[W_t]$,
- e is temporally covered by at least one vertex appearance $(w, t) \in \mathcal{S}[W_t]$.

Example ($\Delta = 4$)

– $\{(c, 2), (c, 3), (c, 6), (c, 8)\}$ is not a sliding Δ-window temporal vertex cover, as edges $cv, cw \in E[W_4]$ are not temporally covered in window W_4.
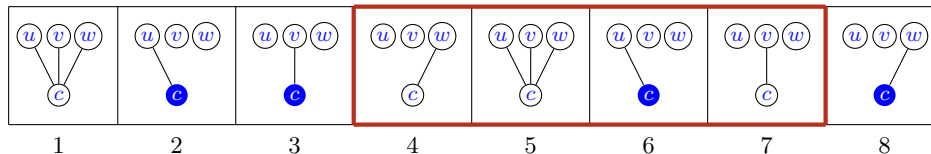
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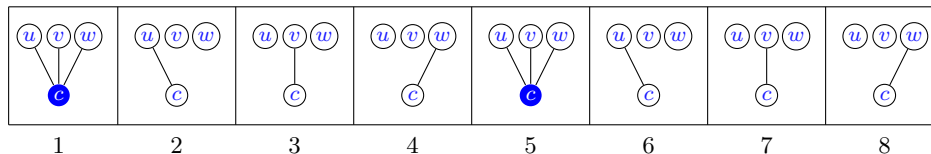

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- $\{(c, 1), (c, 5)\}$ is a sliding Δ-window temporal vertex cover.

SLIDING WINDOW TEMPORAL VERTEX COVER (SW-TVC)

Input: A temporal graph (G, λ) with lifetime T, and an integer $\Delta \leq T$.
Output: A sliding Δ-window temporal vertex cover $\mathcal{S}$ of (G, λ) with the minimum $|\mathcal{S}|$.

Overview

- Basic definitions

- Temporal vertex cover

- Temporal vertex cover with a sliding time window

- Open problems

Temporal Vertex Cover: the star temporal case

> **Lemma**
>
> TVC on *star temporal graphs* is equivalent to SET COVER.

- leafs of the underlying star ↔ ground set of the SET COVER instance
- each snapshot graph ↔ a set in the SET COVER instance

Goal: Choose sets (snapshots) to cover all elements (leafs' edges)

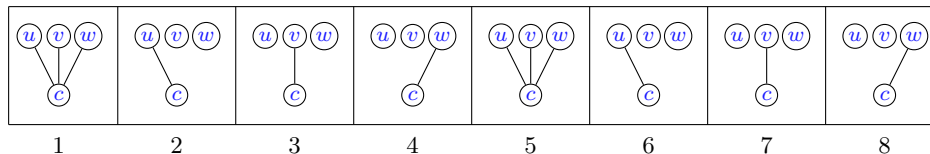**Example:**

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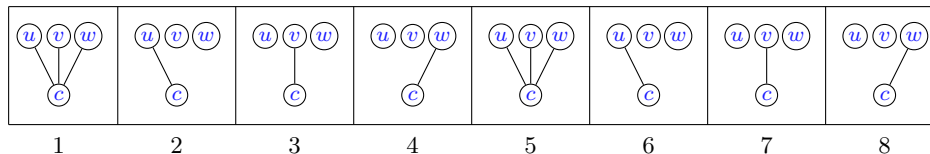
1. Universe: $\{u, v, w\}$
2. Sets: $S_1 = \{u, v, w\}$, $S_2 = \{u\}$, $S_3 = \{v\}$, $S_4 = \{w\}$, ...

Temporal Vertex Cover: the star temporal case

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Consequences:

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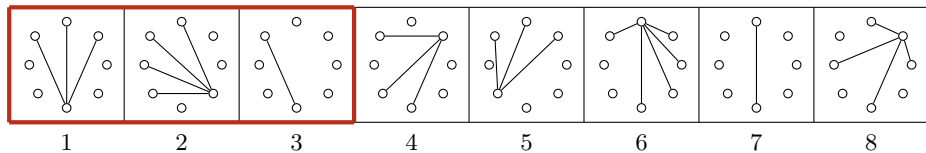TVC *on star temporal graphs is equivalent to* SET COVER.

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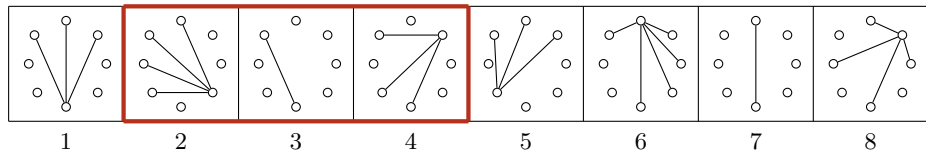TVC *on star temporal graphs is equivalent to* HITTING SET.

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4. TVC on star temporal graphs can be $\ln n$-approximated in polynomial time.
5. For general graphs: $2 \ln n$-approximation algorithm by a similar reduction from TVC to SET COVER

Overview

- Basic definitions

- Alternative models

- Temporal vertex cover

- Temporal vertex cover with a sliding time window

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SW-TVC: **always star** temporal graphs

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1	2	3	4	5	6	7	8

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$\Rightarrow$ we assign a Boolean variable $x_i \in \{0, 1\}$ for the snapshot at time i
- For variables $x_1, x_2, \ldots, x_\Delta$ we define $f(t; x_1, x_2, \ldots, x_\Delta)$ to be the smallest cardinality of a sliding Δ-window temporal vertex cover $\mathcal{S}$ of $(G, \lambda)|_{[1, t+\Delta-1]}$, such that the solution at times $t, t+1, \ldots, t+\Delta-1$ is defined by the variables $x_1, x_2, \ldots, x_\Delta$.

$$f(6; 1, 0, 1)$$

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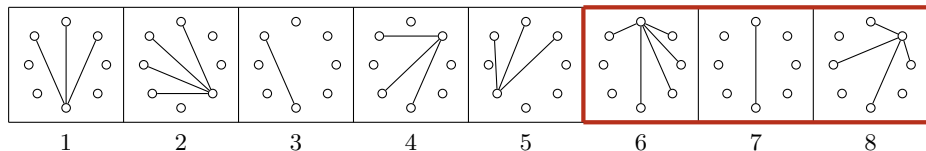
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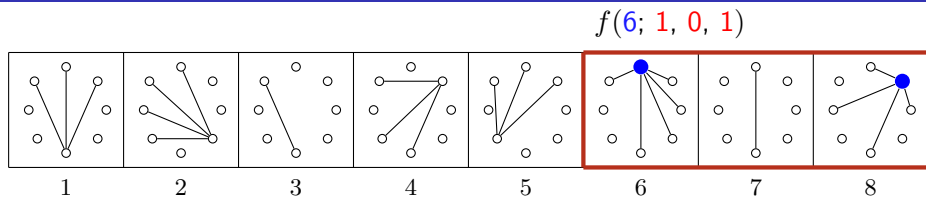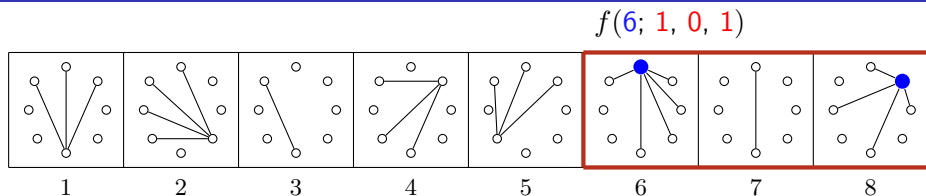- ⇒ we assign a Boolean variable $x_i \in \{0, 1\}$ for the snapshot at time i
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Lemma (dynamic programming)

$$f(t; x_1, x_2, \ldots, x_\Delta) = x_\Delta + \min_{y \in \{0, 1\}} \{f(t-1; y, x_1, x_2, \ldots, x_{\Delta-1})\}$$

SW-TVC: **always star** temporal graphs

Theorem

SW-TVC *on always star temporal graphs can be solved in* $O(T\Delta(n+m) \cdot 2^{\Delta})$ *time.*

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Algorithm SW-TVC on always star temporal graphs

Input: An always star temporal graph (G, λ) with lifetime T and a natural $\Delta \leq T$.
Output: The cardinality of a minimum sliding Δ-window temporal vertex cover in (G, λ).

> **for** $t = 1$ to $T - \Delta + 1$ **do**
> > **for** all $x_1, x_2, \ldots, x_\Delta \in \{0,1\}$ **do**
> > > **if** $\{(c_{t+i-1}, t+i-1) \mid x_i = 1\}$ is a TVC of $(G, \lambda)|_{[t,t+\Delta-1]}$ **then**
> > > > **if** $t = 1$ **then**
> > > > > $f(t; x_1, \ldots, x_\Delta) \leftarrow \sum_{i=1}^{\Delta} x_i$
> > > > **else**
> > > > > $f(t; x_1, \ldots, x_\Delta) \leftarrow x_\Delta + \min_{y \in \{0,1\}} \{f(t-1; y, x_1, \ldots, x_{\Delta-1})\}$
> > > **else**
> > > > $f(t; x_1, \ldots, x_\Delta) \leftarrow \infty$
> **return** $\min_{x_1, \ldots, x_\Delta \in \{0,1\}} \{f(T - \Delta + 1; x_1, \ldots, x_\Delta)\}$

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Main idea:

- for each of the Δ snapshots in the (currently) last Δ-window, we enumerate all 2^n vertex subsets,
- instead of just enumerating over the truth values of Δ Boolean variables ("always star" case)

SW-TVC: the general case

Theorem

SW-TVC *on general temporal graphs can be solved in* $O(T\Delta(n + m) \cdot 2^{n(\Delta+1)})$ *time.*

Algorithm SW-TVC on general temporal graphs

Input: A temporal graph (G, λ) with lifetime T and a natural $\Delta \leq T$.
Output: The smallest cardinality of a sliding Δ-window temporal vertex cover in (G, λ).

1: **for** $t = 1$ to $T - \Delta + 1$ **do**
2:     **for** all $A_1, A_2, \ldots, A_\Delta \subseteq V$ **do**
3:         **if** $\bigcup_{i=1}^{\Delta}(A_i, t + i - 1)$ is a TVC of $(G, \lambda)|_{[t,t+\Delta-1]}$ **then**
4:             **if** $t = 1$ **then**
5:                 $f(t; A_1, \ldots, A_\Delta) \leftarrow \sum_{i=1}^{\Delta} |A_i|$
6:             **else**
7:                 $f(t; A_1, \ldots, A_\Delta) \leftarrow |A_\Delta| + \min_{X \subseteq V} \{f(t - 1; X, A_1, \ldots, A_{\Delta-1})\}$
8:         **else**
9:             $f(t; A_1, \ldots, A_\Delta) \leftarrow \infty$
    return $\min_{A_1, \ldots, A_\Delta \subseteq V} \{f(T - \Delta + 1; A_1, \ldots, A_\Delta)\}$

SW-TVC: Optimality under ETH

Theorem

For any two (arbitrarily growing) functions $f : \mathbb{N} \to \mathbb{N}$ and $g : \mathbb{N} \to \mathbb{N}$, there exists a constant $\varepsilon \in (0,1)$ such that SW-TVC cannot be solved in $f(T) \cdot 2^{\varepsilon n \cdot g(\Delta)}$ time assuming ETH.

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Proof (idea):

- reduction from VERTEX COVER
- $T = \Delta = 2$
- $G_1 = G$; G_2 is an independent set
- given f and g, choose appropriate ε

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Corollary

Our $O(T\Delta(n + m) \cdot 2^{n(\Delta+1)})$-time algorithm is asymptotically almost optimal.

SW-TVC: always bounded vertex cover number temporal graphs

Let $\mathcal{C}_k$ be the class of graphs with the vertex cover number at most k.

Theorem

SW-TVC *on always* $\mathcal{C}_k$ *temporal graphs can be solved in* $O(T\Delta(n+m) \cdot n^{k(\Delta+1)})$ *time.*

Let $\mathcal{C}_k$ be the class of graphs with the vertex cover number at most k.

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Main idea:

- in the optimal solution, the choice at step i is a subset of a minimum vertex cover at this snapshot
- $\Rightarrow$ for each of the Δ last snapshots, enumerate all n^k vertex subsets (candidates for vertex cover at snapshot i)

Δ-TVC

If the parameter Δ (the size of a sliding window) is fixed, we refer to SW-TVC as Δ-TVC (i.e. Δ is a part of the problem name).

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Δ-TVC

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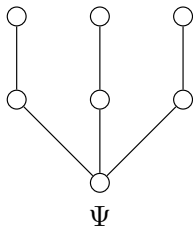
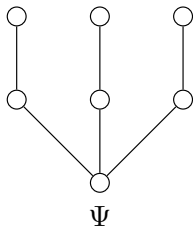## Observation

$(\Delta + 1)$-TVC *is at least as hard as* Δ-TVC.

2-TVC is hard to approximate

Let $\mathcal{X}$ be the class of graphs whose connected components are induced subgraphs of graph Ψ.

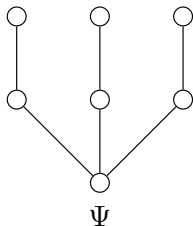

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Theorem

*There is no PTAS for 2-TVC on always $\mathcal{X}$ temporal graphs.*

# 2-TVC is hard to approximate

Theorem

*There is no PTAS for 2-TVC on always $\mathcal{X}$ temporal graphs.*

Proof (sketch):

1. Let $\mathcal{Y}$ be the class of graphs which can be obtained from cubic graphs by subdividing every edge exactly 4 times.
2. There is no PTAS for VERTEX COVER on $\mathcal{Y}$.
3. Reduce VERTEX COVER on $\mathcal{Y}$ to 2-TVC on always $\mathcal{X}$ temporal graphs such that optimal solutions of both problems have same size.
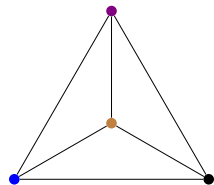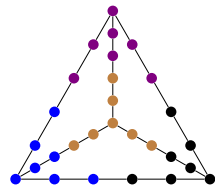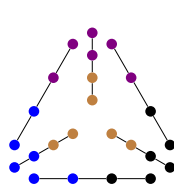

K_4　　　The 4-subdivision of K_4　　　Snapshot G_1　　　Snapshot G_2
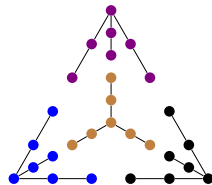
Reduction from SW-TVC to SET COVER.

Reduction from SW-TVC to SET COVER.

1. **The universe:** the set of all pairs $(e, t) \in E \times [T - \Delta + 1]$ such that e appears (and so must be temporally covered) within window W_t.

SW-TVC: approximation algorithms I

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Consequences:

1. $O(\ln n + \ln \Delta)$-approximation
   (every set $C_{v,s}$ has at most $n\Delta$ elements
   $\Rightarrow$ approximation factor $H_{n\Delta} - \frac{1}{2} \approx \ln n + \ln \Delta$)

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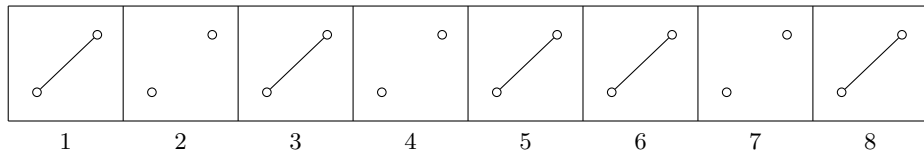
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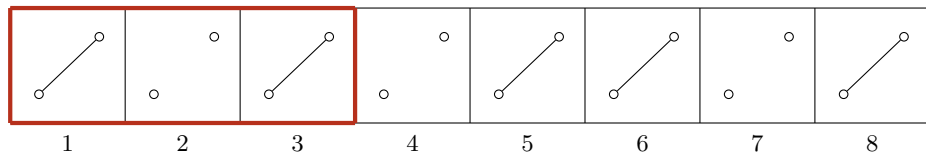
$\Rightarrow$ 2Δ-approximation

Single-edge temporal graph: exact algorithm

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Single-edge temporal graph: **exact algorithm**

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Single-edge temporal graph: exact algorithm

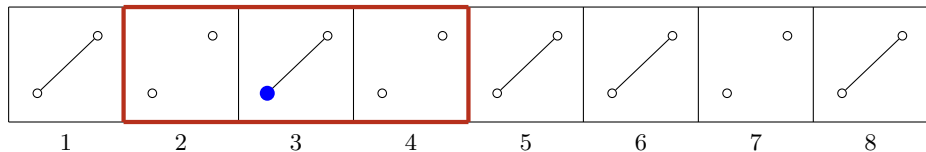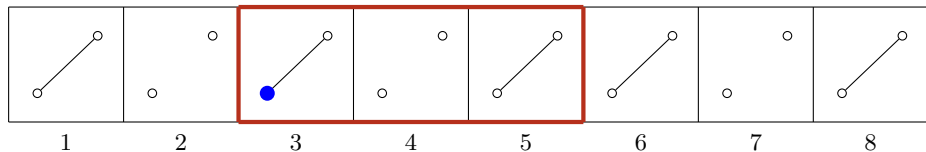**Single-edge temporal graph: exact algorithm**

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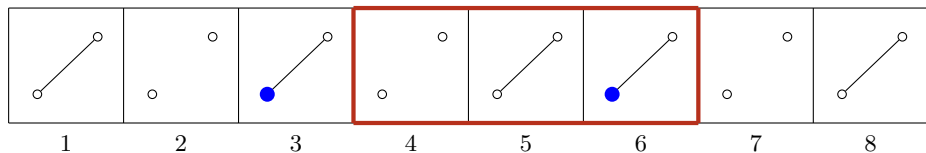**Algorithm** SW-TVC on single-edge temporal graphs

---

Input: A temporal graph (G, λ) of lifetime T with $V(G) = \{u, v\}$; and $\Delta \leq T$.
Output: A minimum-cardinality sliding Δ-window temporal vertex cover $\mathcal{S}$ of (G, λ).

1: $\mathcal{S} \leftarrow \emptyset$
2: $t = 1$
3: **while** $t \leq T - \Delta + 1$ **do**
4:      **if** $\exists\ r \in [t, t + \Delta - 1]$ such that $uv \in E_r$ **then**
5:          choose maximum such r and add (u, r) to $\mathcal{S}$
6:          $t \leftarrow r + 1$
7:      **else**
8:          $t \leftarrow t + 1$
     return $\mathcal{S}$

- greedy algorithm
- linear time

SW-TVC: approximation algorithms II

Always degree at most d temporal graphs: d-approx. algorithm

Main idea:

- solve independently each single-edge subgraph of G
- take the union of the solutions

Always degree at most d temporal graphs: d-approx. algorithm

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Algorithm d-approximation of SW-TVC on always degree at most d temporal graphs

---

Input: An always degree at most d temporal graph (G, λ) of lifetime T, and $\Delta \leq T$.
Output: A sliding Δ-window temporal vertex cover $\mathcal{S}$ of (G, λ).
 1: **for** every edge $uv \in E(G)$ **do**
 2:     Compute an optimal solution $\mathcal{S}_{uv}$ of the problem for $(G[\{u, v\}], \lambda)]$
 3:     $\mathcal{S} \leftarrow \mathcal{S} \cup \mathcal{S}_{uv}$
    return $\mathcal{S}$

SW-TVC: approximation algorithms II

Always degree at most d temporal graphs: d-approx. algorithm

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Lemma

The above algorithm is a $O(mT)$-time d-approximation algorithm for SW-TVC on always degree at most d temporal graphs.

SW-TVC: approximation algorithms II

Always degree at most d temporal graphs: d-approx. algorithm

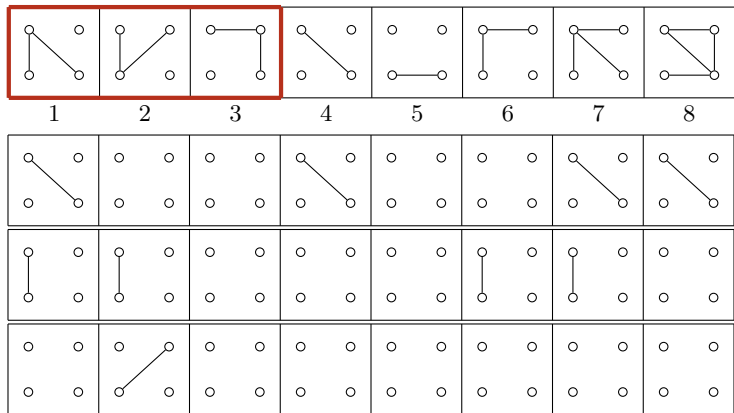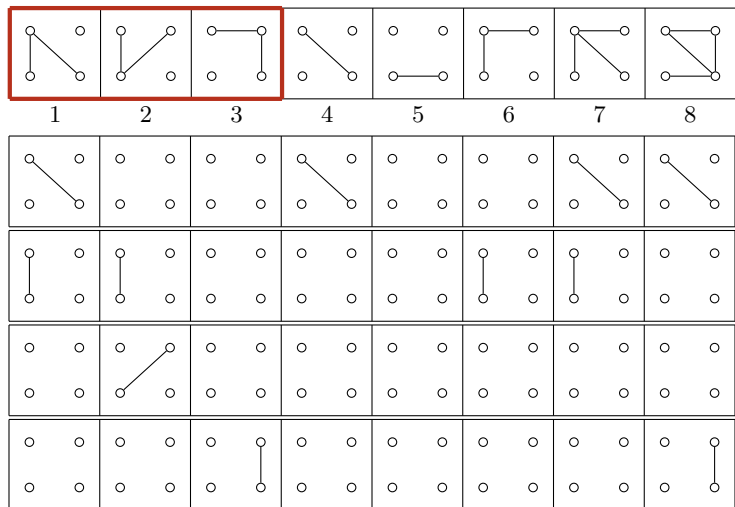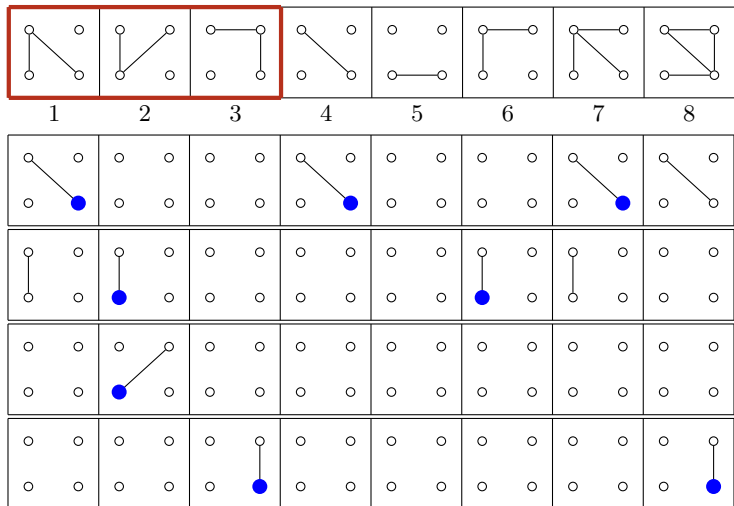**Algorithm** d-approximation of SW-TVC on always degree at most d temporal graphs

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Lemma

The above algorithm is a $O(mT)$-time d-approximation algorithm for SW-TVC on always degree at most d temporal graphs.

Corollary

SW-TVC *can be optimally solved in $O(mT)$ time on the class of always degree at most 1 (matching) temporal graphs.*

Overview

- Basic definitions

- Alternative models

- Temporal vertex cover

- Temporal vertex cover with a sliding time window

- Open problems

Open Problems

Problem 1

Determine the complexity status of Δ-TVC on degree at most 2 temporal graphs.

1. Δ-TVC on always degree at most 1 can be solved in linear time.

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Problem 3

Can Δ-TVC on always degree at most d temporal graphs be approximated within a factor better than d?

Problem 4

FPT results for Δ-TVC with respect to Δ on general temporal graphs?

Open Problems

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FPT results for Δ-TVC with respect to Δ on general temporal graphs?

1. SW-TVC on general temporal graphs can be solved in $O(T\Delta(n+m) \cdot 2^{n(\Delta+1)})$ time.

Open Problems

Problem 4

FPT results for Δ-TVC with respect to Δ on general temporal graphs?

1. SW-TVC on general temporal graphs can be solved in $O(T\Delta(n+m) \cdot 2^{n(\Delta+1)})$ time.

Problem 5

FPT results for SW-TVC with respect to the maximum vertex cover number k of snapshots?

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Problem 5

FPT results for SW-TVC with respect to the maximum vertex cover number k of snapshots?

1. SW-TVC on always vertex cover number at most k temporal graphs can be solved in $O(T\Delta(n+m) \cdot n^{k(\Delta+1)})$ time.

Thank you!