Parameterized Cops'n'Robbers on Edge-Periodic Temporal Graphs

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examples by Nils Morawietz, used with approval

Algorithmic Aspects of Temporal Graphs IV
Satelite Workshop of ICALP 2021

Intro I – Cops and Robber Games

Setup

1. place k "cops"
2. place 1 "robber"

Turns

1. each cop moves along ≤ 1 edge
2. if any cop meets the robber
   $\rightsquigarrow$ cops win
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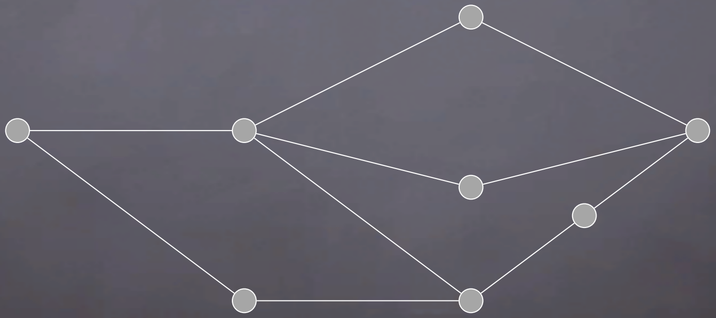

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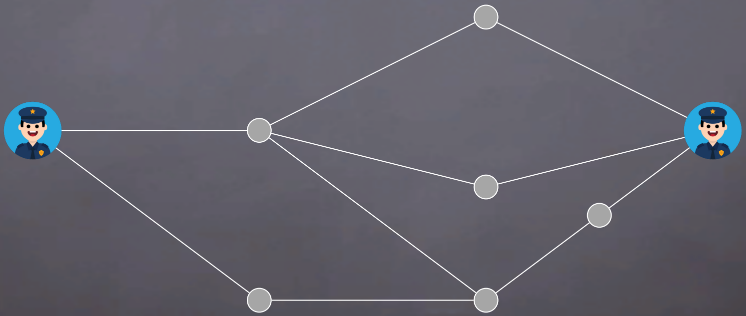

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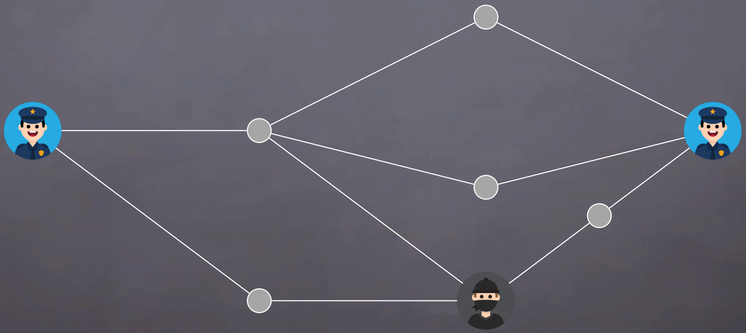

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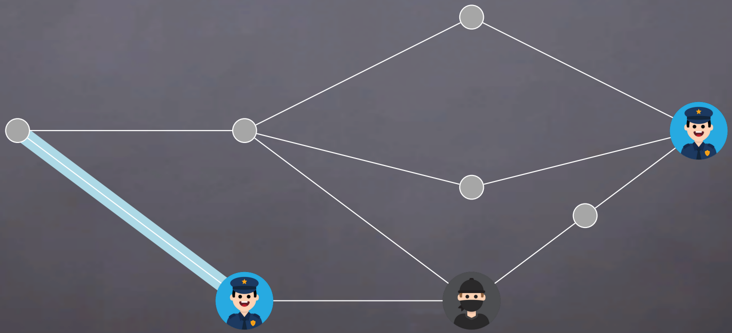

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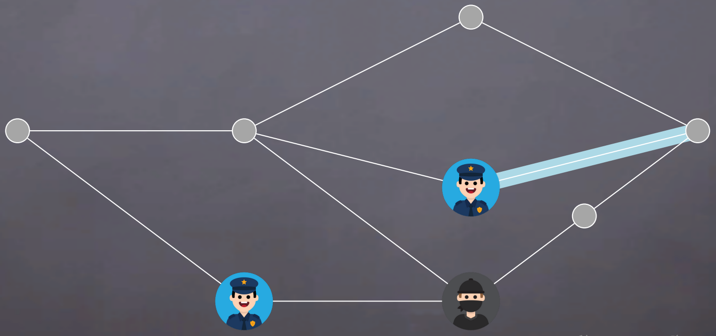

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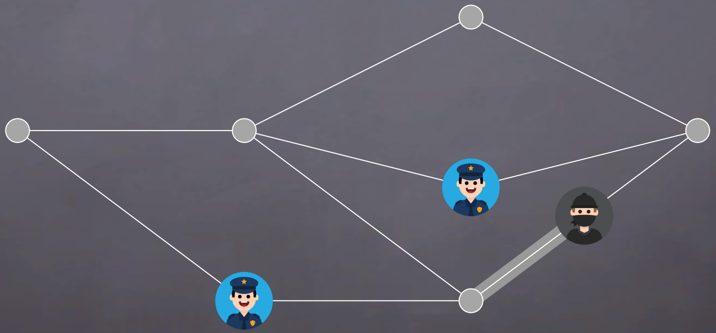

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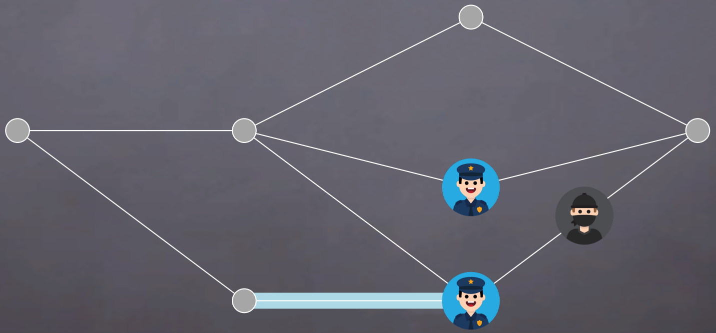

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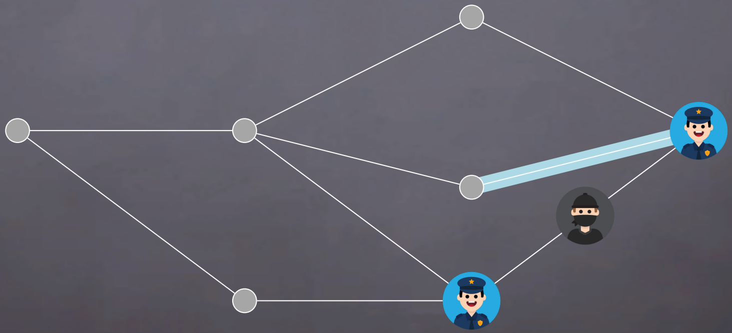

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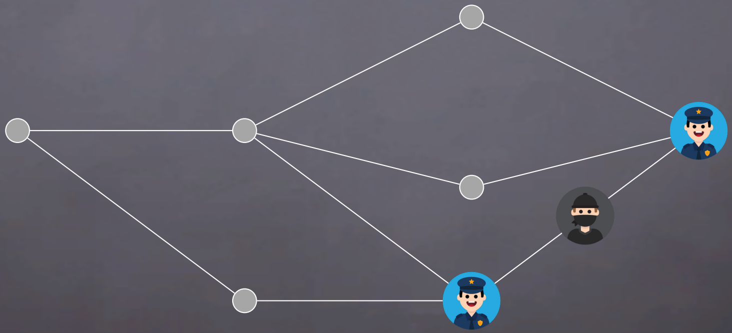

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Notes

- variants model important concepts in graph theory: (directed) tree- & path-, DAG- and Kelly width, etc
- k cops can win?
  $\rightsquigarrow$ NP-hard in general
  $\rightsquigarrow$ $k^{O(k^3)}n$ time [Bodlaender '96]
- in the following: $k = 1$

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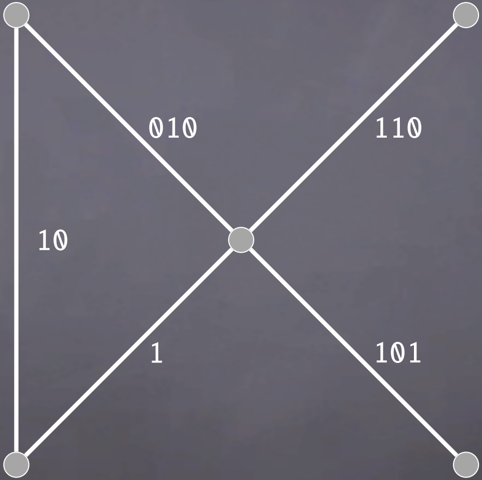
edge e_i exists in $G(t) \Leftrightarrow x_i[t \bmod |x_i|] = 1$



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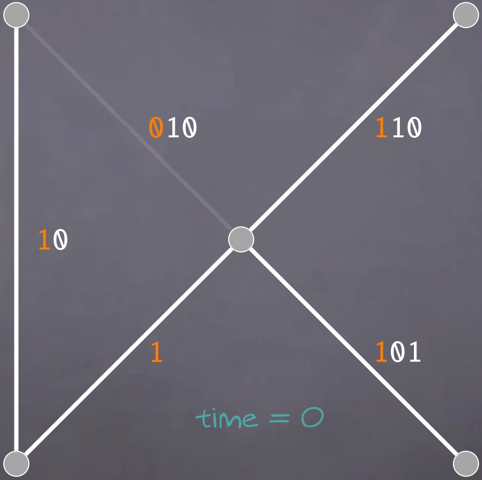

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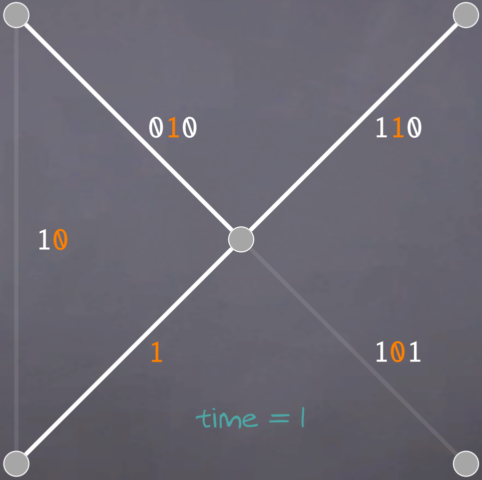

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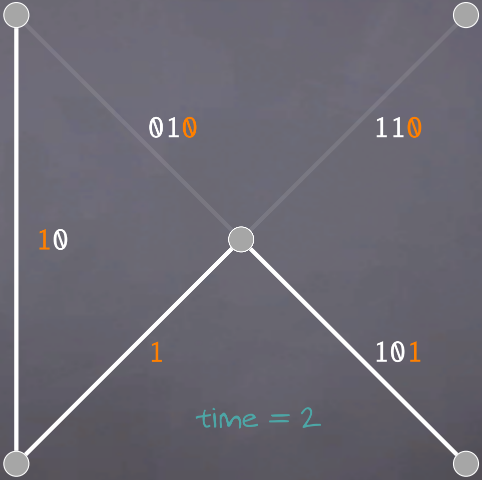

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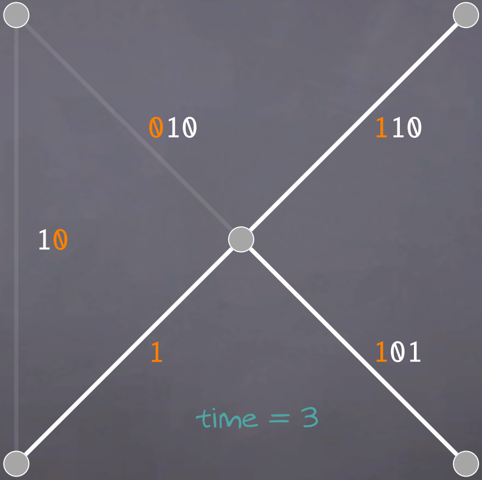

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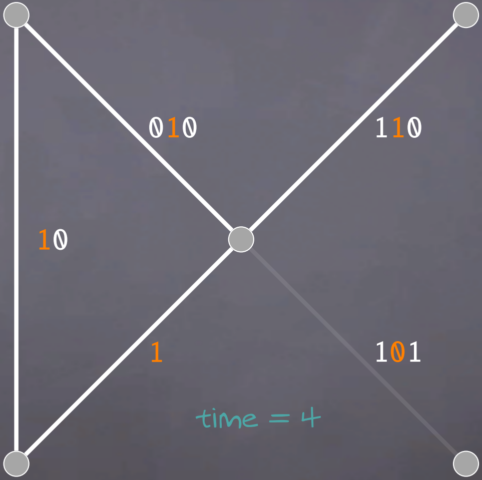

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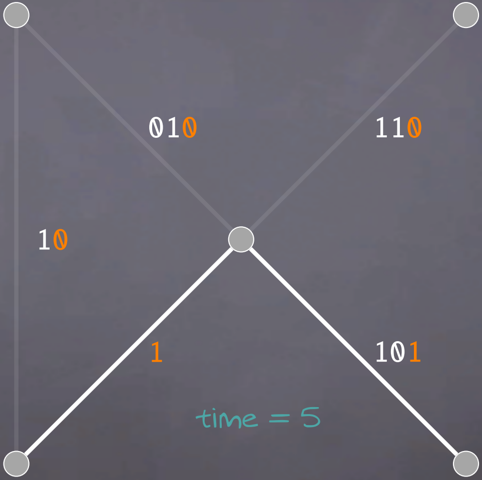

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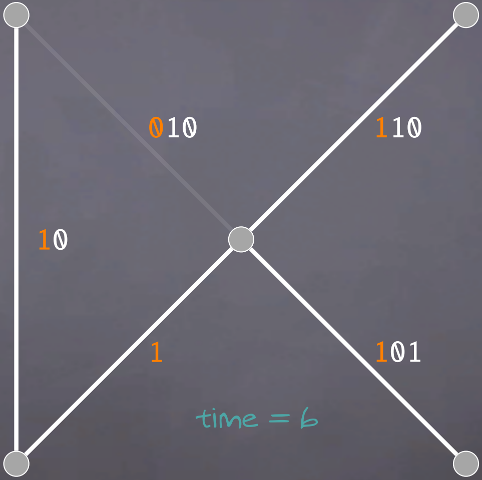

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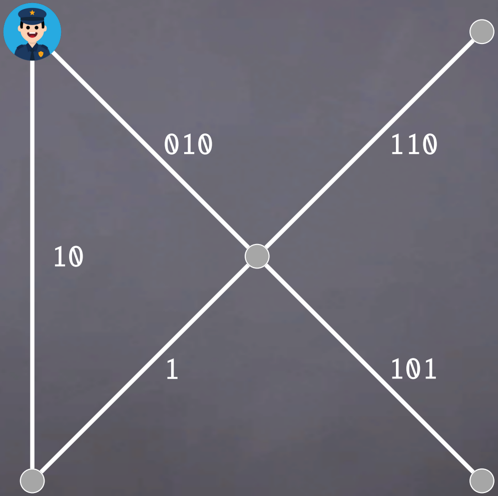
Note: periodic in lcm of all sequence-lengths

Intro I + II – C'n'R on Periodic Graphs



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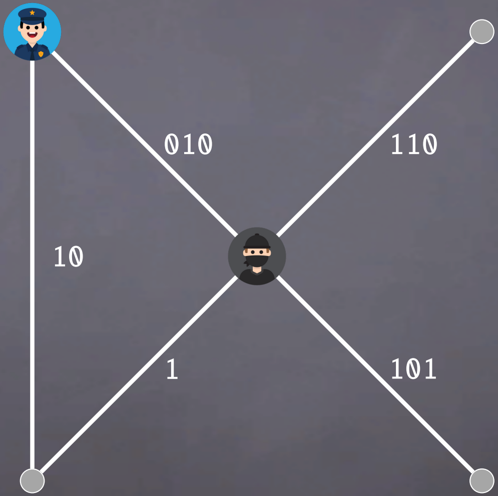

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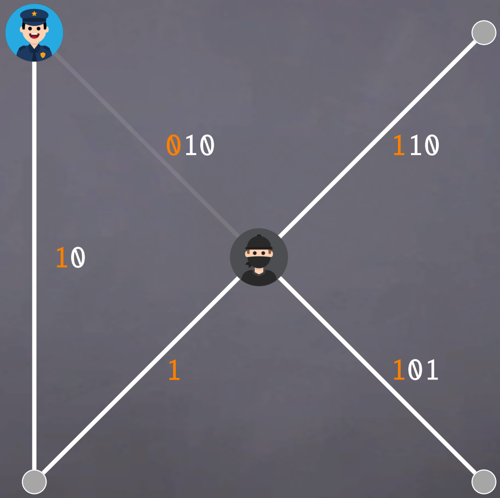

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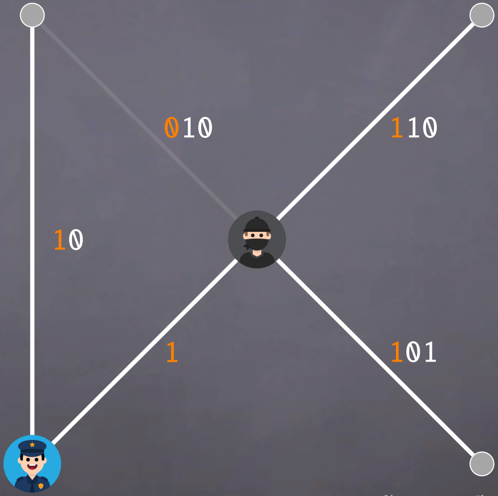

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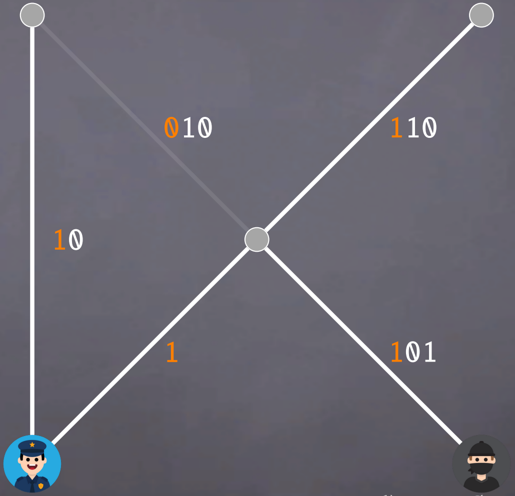

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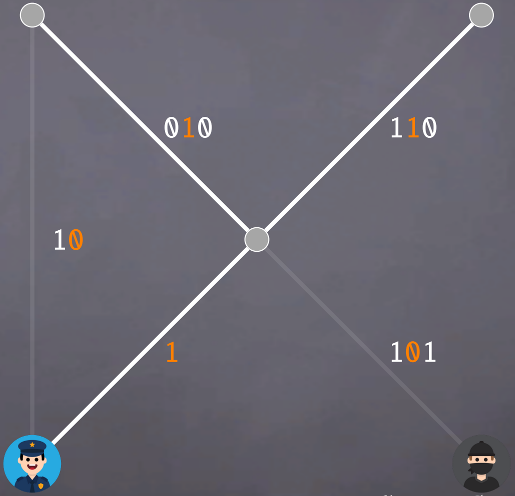

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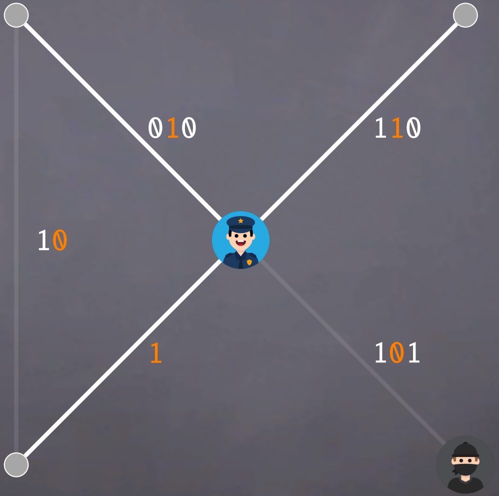

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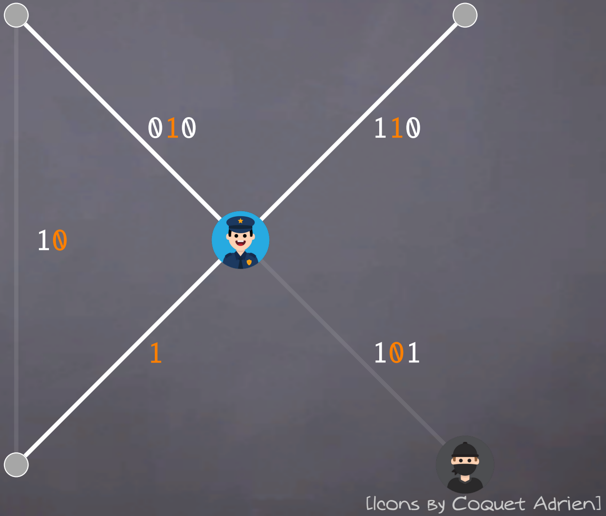

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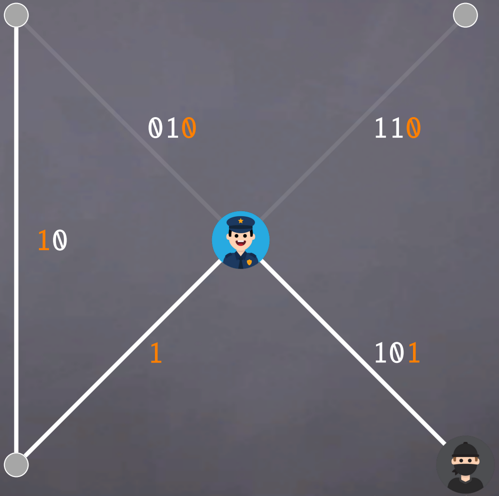

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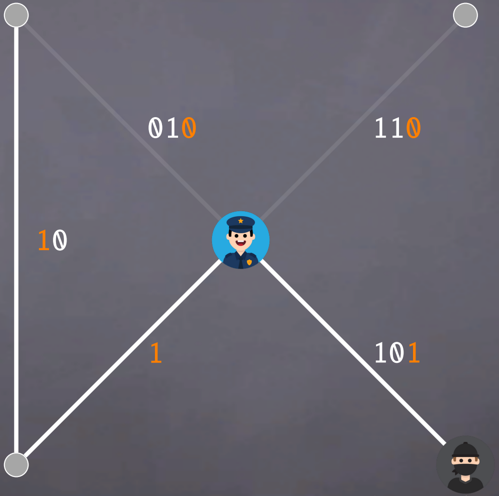

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Note: introduced by Erlebach & Spooner [SOFSEM'20]

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1101110111011101110111011101...
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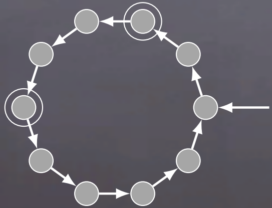

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- Periodic Cop & Robber:
  - W[1]-hard for $|G|$ even on (directed) cycles and $K_{2,n}$
  - W[1]-hard for . . .
  - in PSPACE [Morawietz & Wolf '21]
- characterization of robber-win periodic cycles by length

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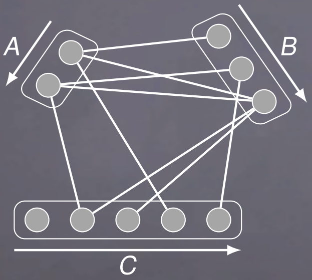

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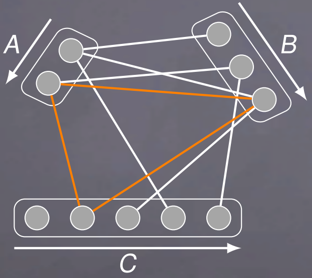

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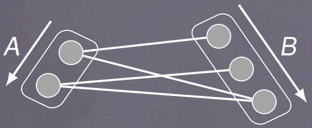

$\overbrace{??????}^{|A|\cdot|B|}$

0 at position $p \Leftrightarrow$
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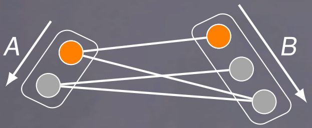

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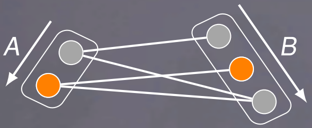

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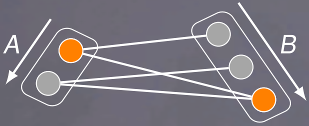

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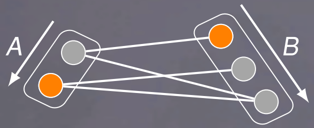

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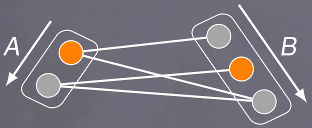

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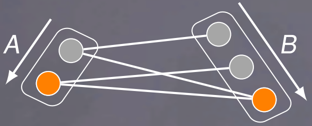

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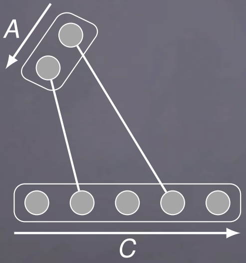

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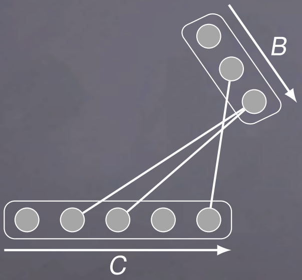

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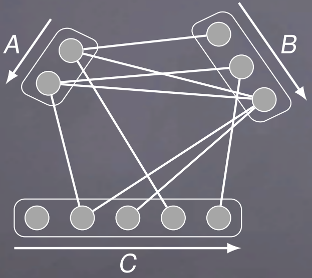


$\overbrace{000110}^{|A|\cdot|B|}$000110000110000110000110

101111110110111111011011111101

110101111110111110101111110111

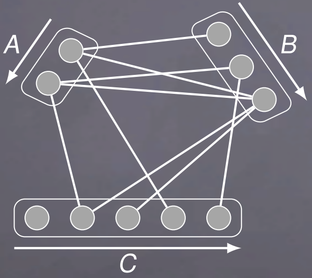
0 at position p $\Leftrightarrow$
edge $\{a_{p \bmod |A|}, b_{p \bmod |B|}\}$

Periodic Alignment and Clique

Multicolored Clique

Input: k-partite graph H

Question: Does H contain a k-clique?



$\overbrace{000110}^{|A|\cdot|B|}000110000110000110000110$

101111110110111111011011111101

110101111110111110101111110111

0 at position $p \Leftrightarrow$ edge $\{a_{p \bmod |A|}, b_{p \bmod |B|}\}$

Periodic Alignment and Clique

Multicolored Clique

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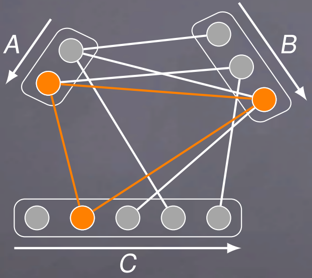

$|A|\cdot|B|$

000110000110000110000110000110
101111110110111111011011111101
110101111110111110101111110111

0 at position p $\Leftrightarrow$
edge $\{a_{p \bmod |A|}, b_{p \bmod |B|}\}$

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Multicolored Clique

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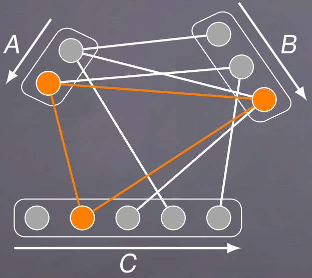

$\overbrace{}^{|A|\cdot|B|}$

000110000110000110000110000110
101111110110111111011011111101
110101111110111110101111110111

0 at position p ⇔ edge $\{a_{p \bmod |A|}, b_{p \bmod |B|}\}$

Theorem

Periodic Alignment ≥ MC Clique

Periodic Alignment and Clique

Multicolored Clique

Input: k-partite graph H

Question: Does H contain a k-clique?

$$k \begin{cases} 001 \\ 1001 \\ 111100 \end{cases}$$

Periodic Alignment and Clique

Multicolored Clique

Input: k-partite graph H

Question: Does H contain a k-clique?

$$k \begin{cases} 001 \\ 1001 \\ 111100 \end{cases}$$

edge $\{i, j\} \Leftrightarrow$
pos i in A and j in B eventually overlap

Periodic Alignment and Clique

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Periodic Alignment and Clique

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$$k \begin{cases} 001 \\ 1001 \\ 111100 \end{cases}$$

111100

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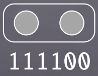

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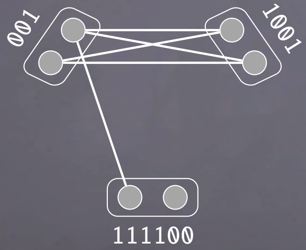

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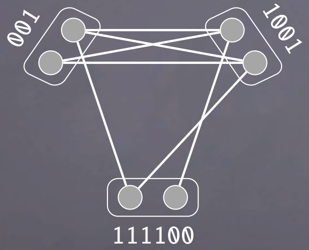

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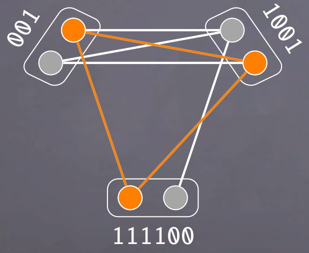

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Periodic Alignment and Clique

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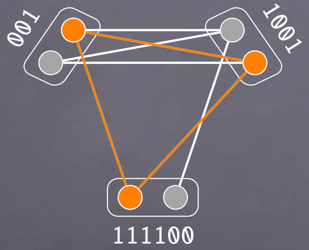

k {
001001001001001001001001001001010...
100110011001100110011001100110011001...
111100111100111100111100111100...
}

edge $\{i, j\} \Leftrightarrow$
pos i in A and j in B eventually overlap $\Leftrightarrow$
$\exists p$ s.t. $i = p \bmod |A|$ and $j = p \bmod |B| \Leftrightarrow$
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Periodic Alignment and Clique

Multicolored Clique

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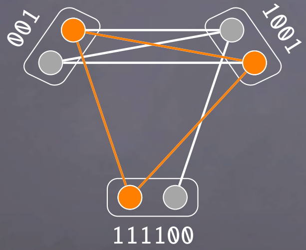


k:

```
001001001000100100100100100100...
100110011000110011001100110011001...
111100111100111100111100111100001111...
```

edge $\{i, j\} \Leftrightarrow$

pos i in A and j in B eventually overlap $\Leftrightarrow$

$\exists p$ s.t. $i = p \bmod |A|$ and $j = p \bmod |B|$ $\Leftrightarrow$

$i \equiv j \mod \gcd(|A|, |B|)$

Theorem

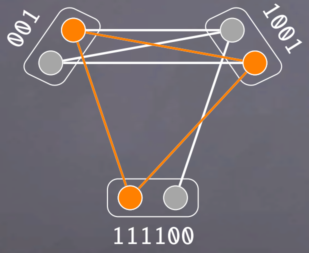
Periodic Alignment $\leq$ MC Clique

Periodic Alignment and Clique

Multicolored Clique

Input: k-partite graph H

Question: Does H contain a k-clique?



k { 00100100100100100100100100010...
1001100110011001100110011001...
1111001111001111001111001111... }

edge $\{i, j\} \Leftrightarrow$
pos i in A and j in B eventually overlap $\Leftrightarrow$
$\exists p$ s.t. $i = p \bmod |A|$ and $j = p \bmod |B| \Leftrightarrow$
$i \equiv j \mod \gcd(|A|, |B|)$

Note: $G[A \cup B]$ decomposes into $\gcd(|A|, |B|)$ bicliques
$\rightsquigarrow$ Periodic Alignment solved in $\gcd^{k^2} \cdot n^{O(1)}$

Periodic Alignment and Clique

Multicolored Clique

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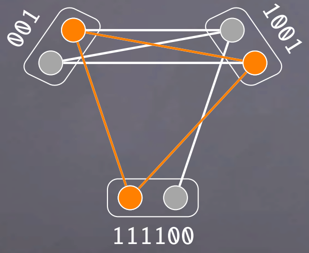
Question: Does H contain a k-clique?



$$k \begin{cases} 001001001001001001001001001001\ldots \\ 1001100110011001100110011001\ldots \\ 111100111100111100111100111\ldots \end{cases}$$

edge $\{i, j\}$ $\Leftrightarrow$
pos i in A and j in B eventually overlap $\Leftrightarrow$
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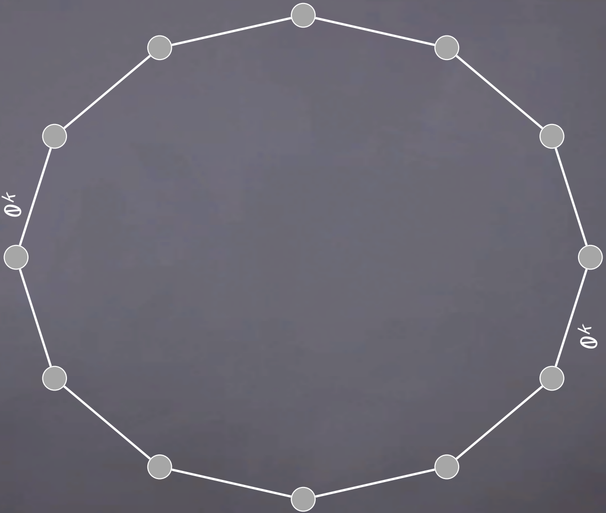
Note: $G[A \cup B]$ decomposes into $\gcd(|A|, |B|)$ bicliques

$\rightsquigarrow$ Periodic Alignment solved in $\gcd^{k^2} \cdot n^{O(1)}$

$\rightsquigarrow$ MC Clique remains $W[1]$-hard even if edges between each partition-pair decompose into disjoint bicliques

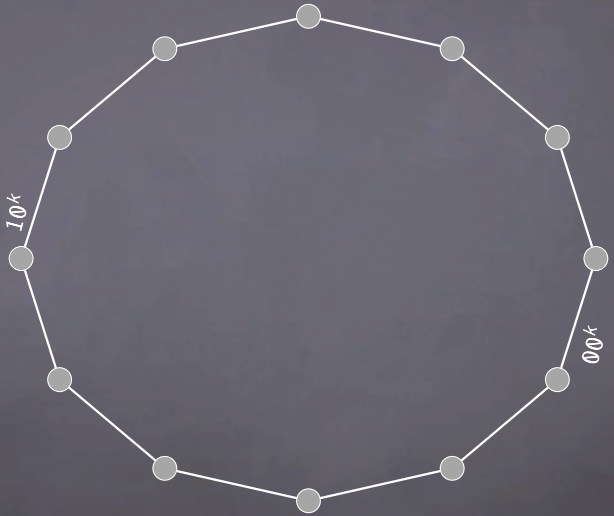
Periodic Alignment and the C'n'R Game



top & bottom e_i: $x_i[0]^k$

[Icons by Coquet Adrien]

Periodic Alignment and the C'n'R Game



top & bottom e_i: $0 \cdot x_i[0]^k$

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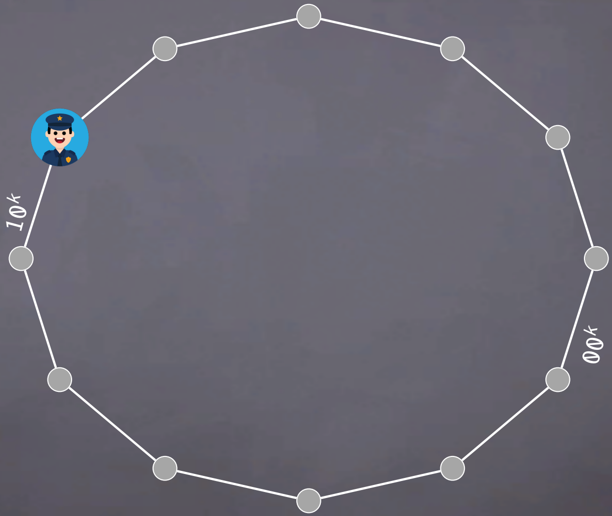
Periodic Alignment and the C'n'R Game



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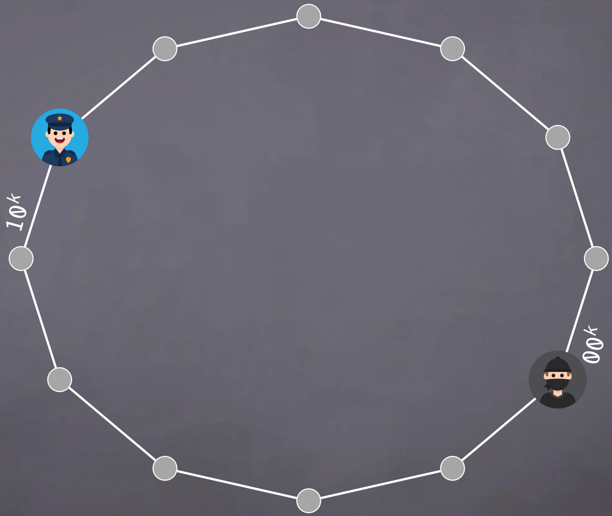

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Periodic Alignment and the C'n'R Game

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Periodic Alignment and the C'n'R Game

top & bottom e_i: $0 \cdot x_i[0]^k$

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Periodic Alignment and the C'n'R Game

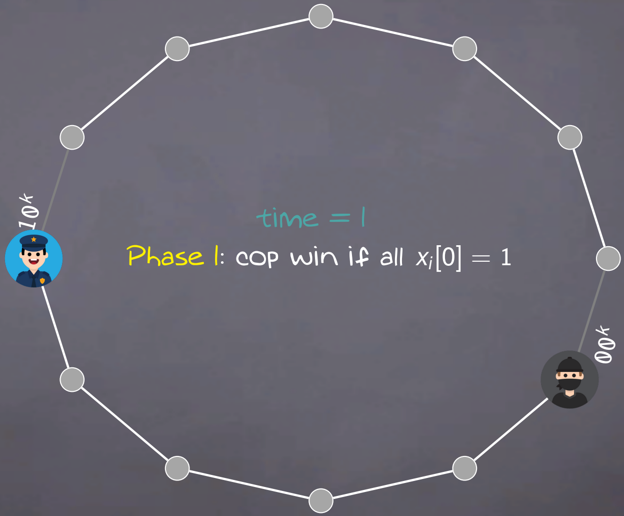
time = 0

Phase I: cop win if all $x_i[0] = 1$

top & bottom e_i: $0 \cdot x_i[0]^k$

[Icons by Coquet Adrien]

Periodic Alignment and the C'n'R Game



time = 1

Phase 1: cop win if all $x_i[0] = 1$

top & bottom e_i: $0 \cdot x_i[0]^k$

[Icons by Coquet Adrien]

Periodic Alignment and the C'n'R Game

top & bottom e_i: $0 \cdot x_i[0]^k$

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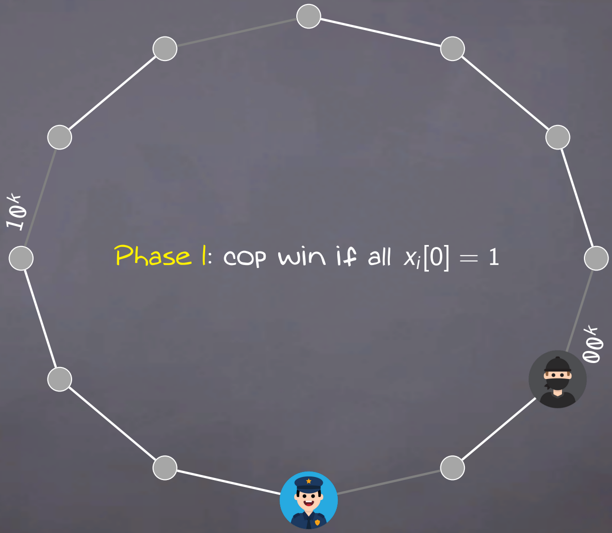
Periodic Alignment and the C'n'R Game



top & bottom e_i: $0 \cdot x_i[0]^k$

[Icons by Coquet Adrien]

Periodic Alignment and the C'n'R Game

top & bottom e_i: $0 \cdot x_i[0]^k$

[Icons by Coquet Adrien]

Periodic Alignment and the C'n'R Game

top & bottom e_i: $0 \cdot x_i[0]^k \cdot 0$

[Icons by Coquet Adrien]

Periodic Alignment and the C'n'R Game

top & bottom e_i: $0 \cdot x_i[0]^k \cdot 0$

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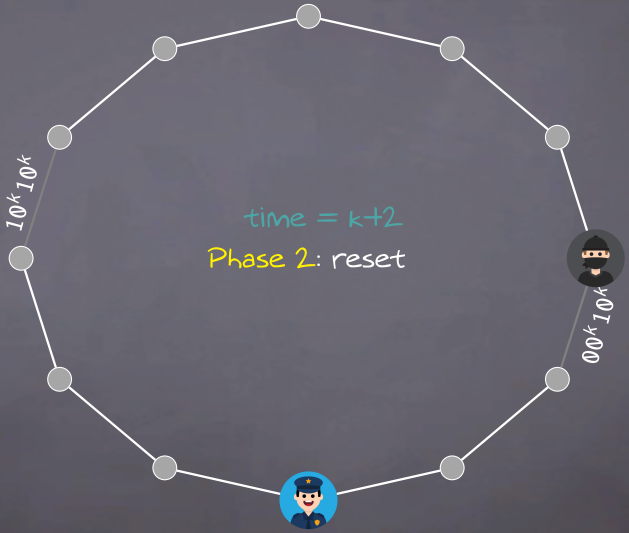
Periodic Alignment and the C'n'R Game



top & bottom e_i: $0\cdot x_i[0]^k\cdot 01^k$

[Icons by Coquet Adrien]

Periodic Alignment and the C'n'R Game

top & bottom e_i: $0 \cdot x_i[0]^k \cdot 01^k$

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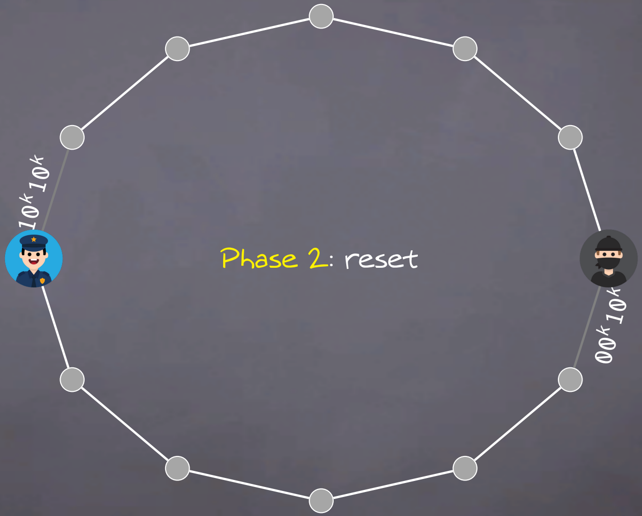

top & bottom e_i: $0 \cdot x_i[0]^k \cdot 01^k$

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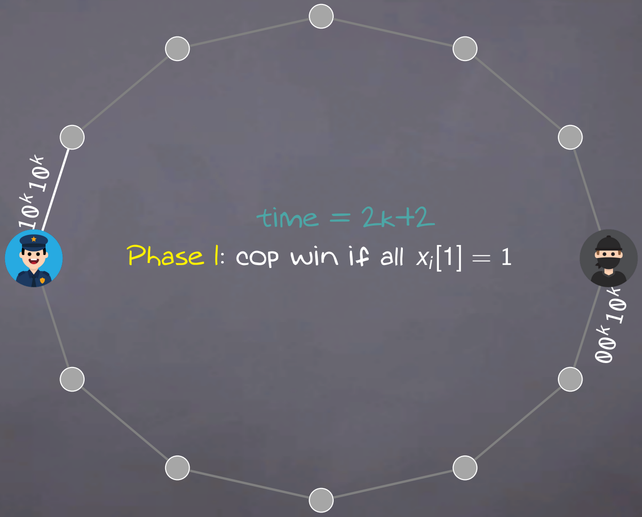
Periodic Alignment and the C'n'R Game



top & bottom e_i: $0 \cdot x_i[0]^k \cdot 01^k 0 \cdot x_i[1]^k \ldots$

[Icons by Coquet Adrien]

Periodic Alignment and the C'n'R Game

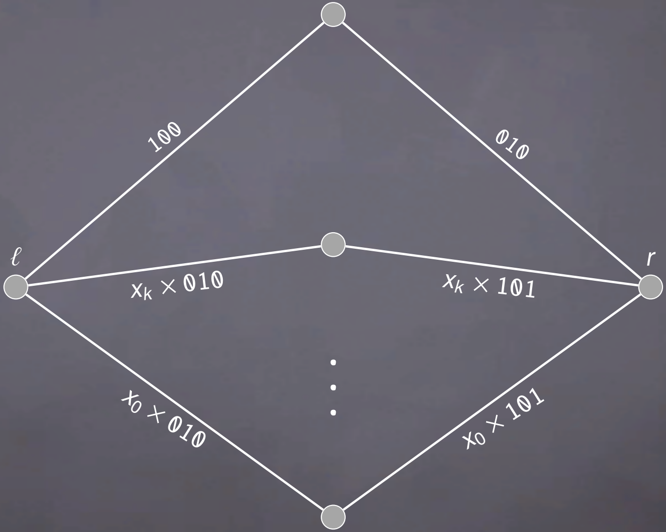
time = 2k+2

Phase I: cop win if all $x_i[1] = 1$

top & bottom e_i: $0 \cdot x_i[0]^k \cdot 01^k 0 \cdot x_i[1]^k \ldots$

[Icons by Coquet Adrien]

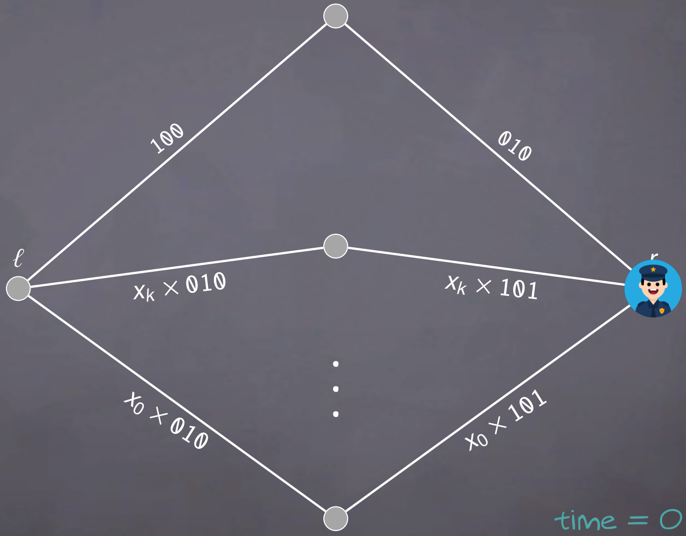
Periodic Alignment and the C'n'R Game



$1101 \times 101 = 101 \ \ 101 \ \ 000 \ \ 101$

[Icons By Coquet Adrien]

Periodic Alignment and the C'n'R Game



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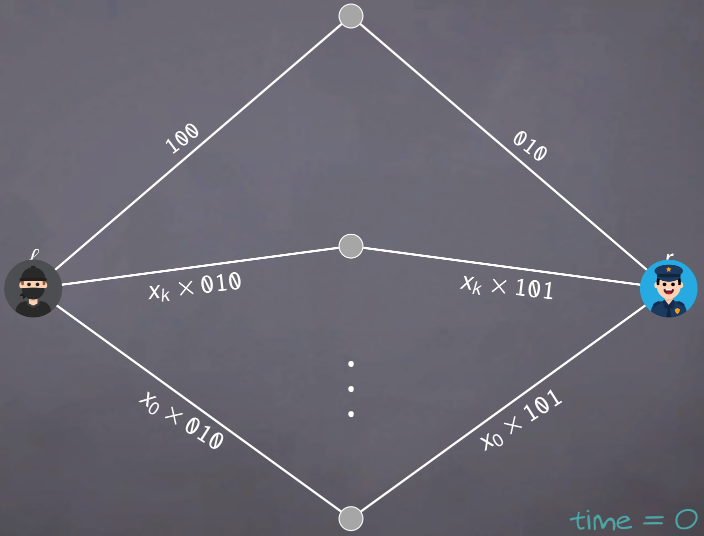

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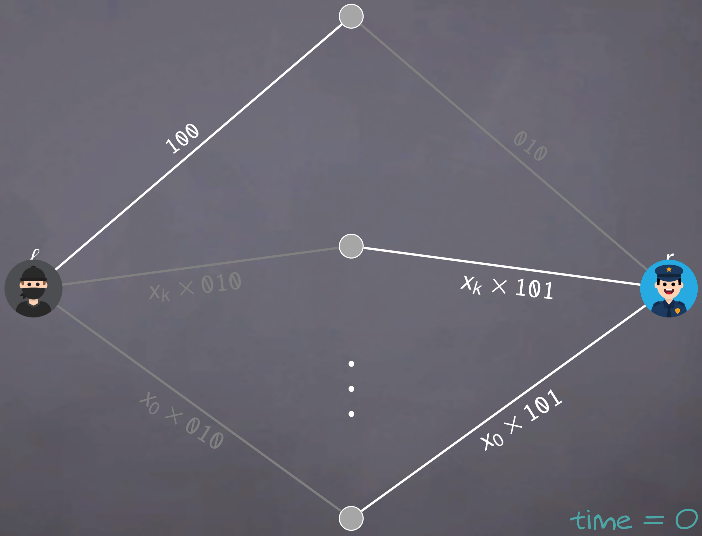

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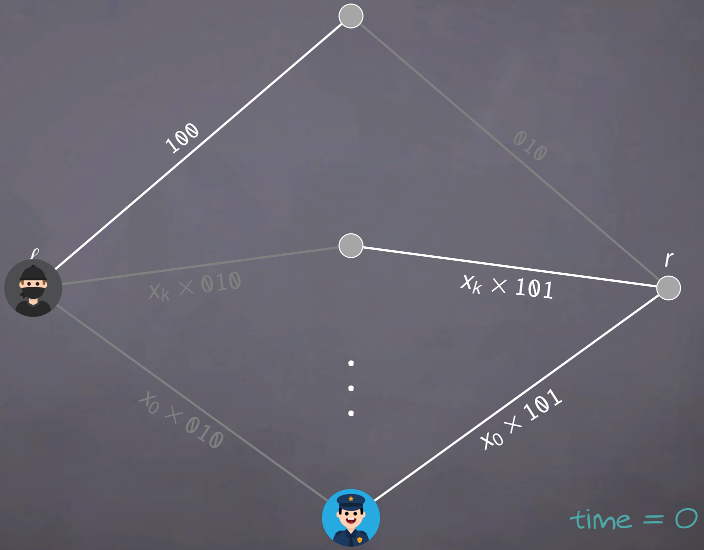

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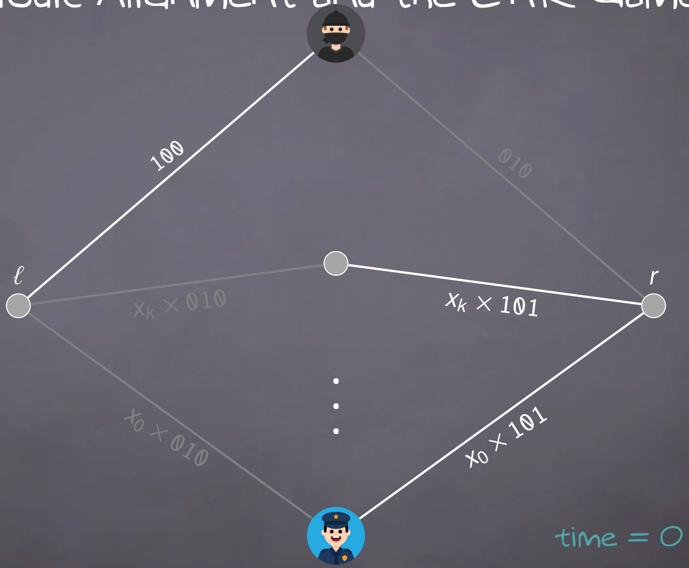

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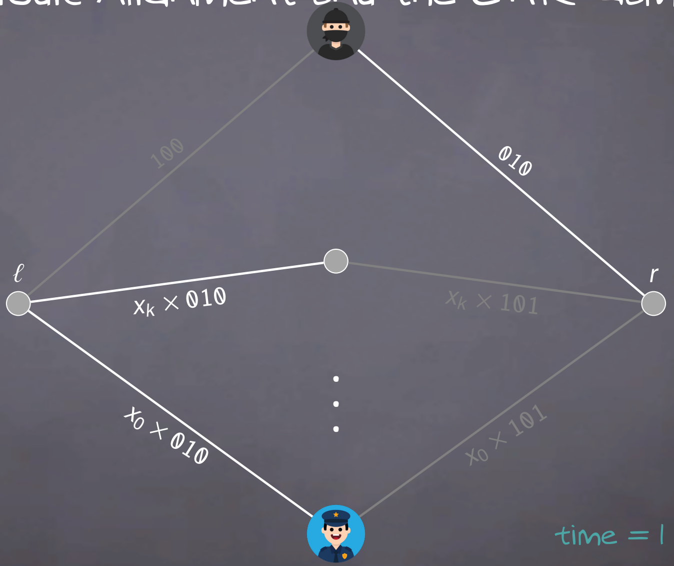

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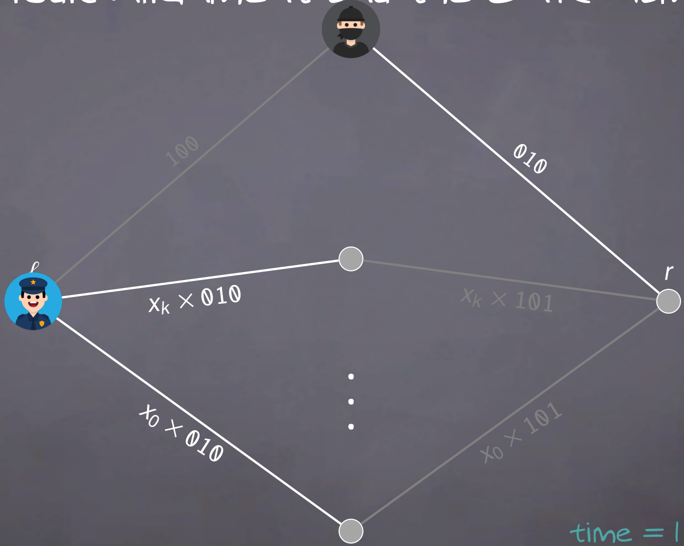

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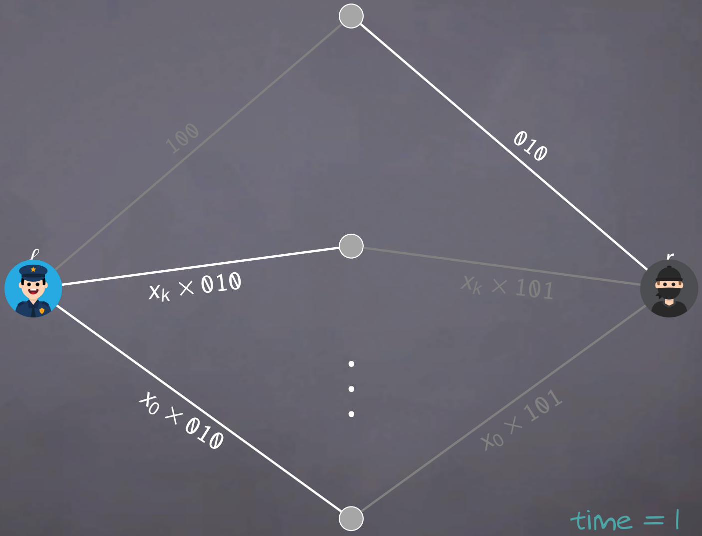

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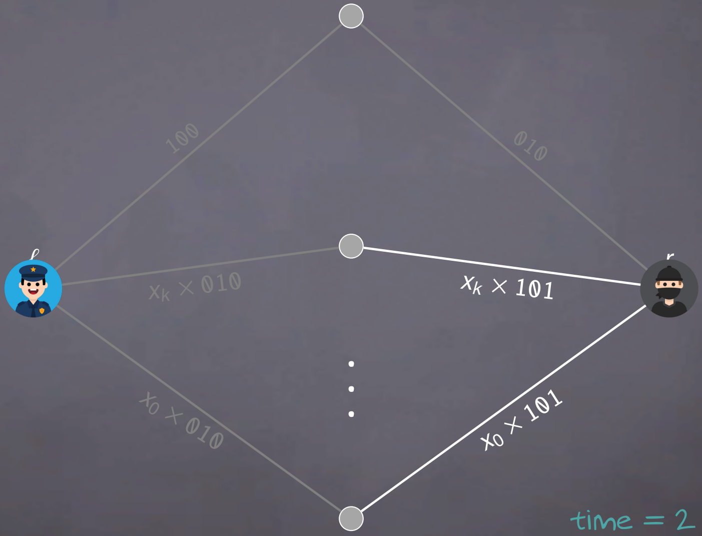

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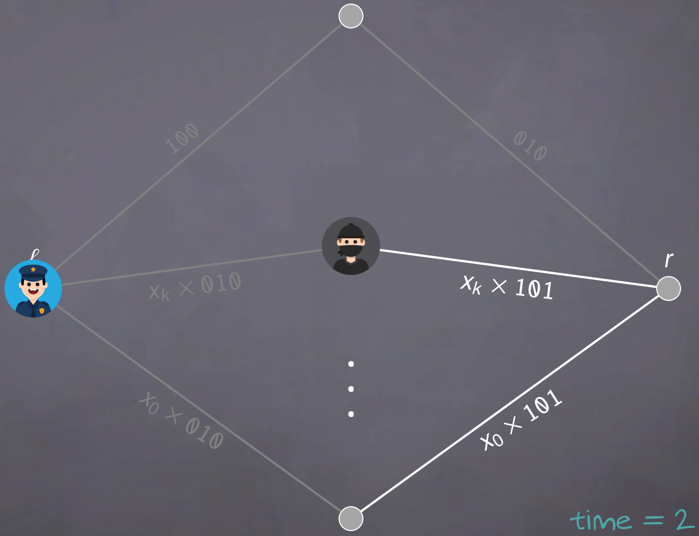

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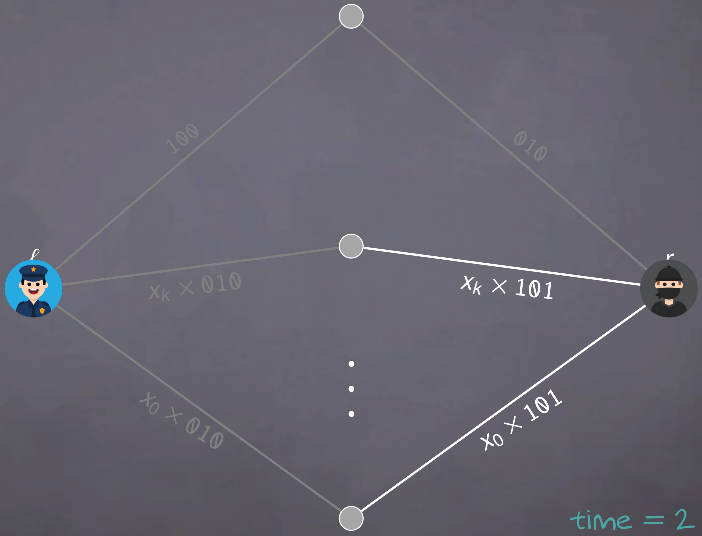

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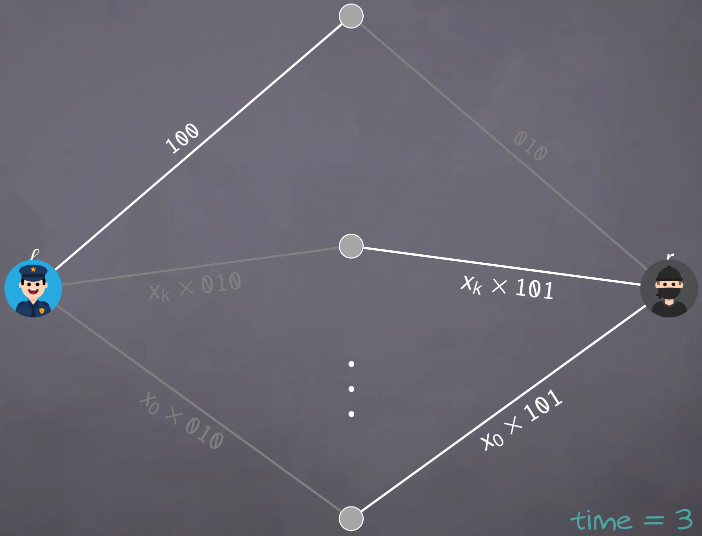

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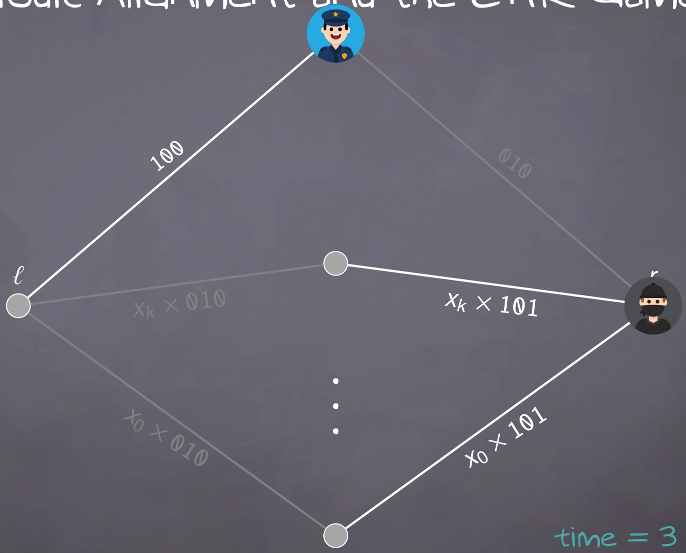
Periodic Alignment and the C'n'R Game



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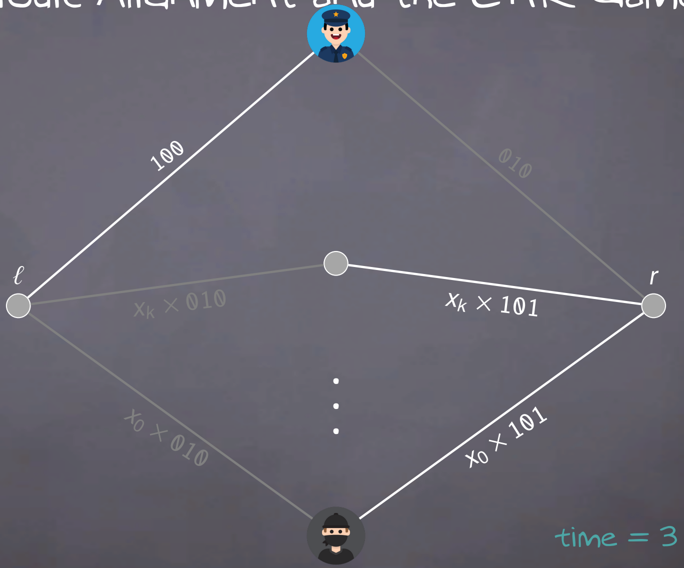
[Icons by Coquet Adrien]

Periodic Alignment and the C'n'R Game



$1101 \times 101 = 101\ \ 101\ \ 000\ \ 101$

[Icons by Coquet Adrien]

Conclusion

Periodic Alignment

Input: seq. X over $\{0, 1\}$

Question: $\exists p$ s.t. $\forall x \in X$ $x[p]^{\circ} = 0$

Summary

- Periodic Alignment = Tally Intersection = MCClique*: $W[1]$-hard wrt. $|X|$

Conclusion

Periodic Partial Alignment

Input: seq. X over $\{0, 1\}$, int ℓ

Question: $\exists$ size-ℓ $Y \subseteq X$ $\exists p$ s.t. $\forall x \in Y$ $x[p]^\circ = 0$

Summary

- Periodic Alignment = Tally Intersection = MCClique*:
  W[1]-hard wrt. $|X|$
- Periodic Partial Alignment:
  W[1]-hard wrt. $|X|$ even if only one 0 per seq.
  FPT wrt. overall #runs
  FPT wrt. (max. pairwise gcd of seq. lengths) + $|X|$

Conclusion

Periodic Cop & Robber

Input: edge-periodic graph G

Question: Is G cop-win?

Summary

- Periodic Alignment = Tally Intersection = MCClique*:
  W[1]-hard wrt. $|X|$
- Periodic Partial Alignment:
  W[1]-hard wrt. $|X|$ even if only one 0 per seq.
  FPT wrt. overall #runs
  FPT wrt. (max. pairwise gcd of seq. lengths) + $|X|$

Conclusion

Periodic Cop & Robber

Input: edge-periodic graph G

Question: Is G cop-win?

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  W[1]-hard wrt. $|X|$ even if only one 0 per seq.
  FPT wrt. overall #runs
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- Periodic Cop & Robber:
  W[1]-hard wrt. $|G|$ if $G = C_{2k+2}$ & no $G(i)$ is cyclic
  W[1]-hard wrt. $|G|$ if $G = K_{2,2k+1}$ & each $G(i)$ star + edge
  W[1]-hard wrt. $|G|$ if G biplanar and $\forall i \, |E(G(i))| \leq 2$
  W[1]-hard wrt. #len + vc# if G biplanar, $\forall i \, |E(G(i))| \leq 2$ & each seq only one 1

  solvable in $O(\text{lcm} \cdot n^3)$

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Open Questions

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Open Questions

- Periodic Alignment:
  find good parameterizations
  kernelization?
- Periodic Cop & Robber:
  $\in$ NP? (known: PSPACE) [Morawietz & Wolf '21]
  hardness if each $G(i)$ is connected?
  variants: invisible robber? fast cop(s)? >1 robber?


Thank
You