Temporal Reachability Minimization: Delaying vs. Deleting

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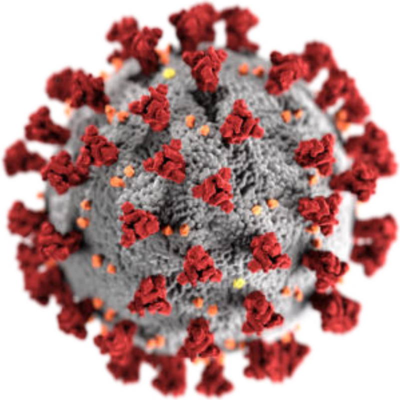
Algorithmic Aspects of Temporal Graphs IV

Extended abstract to appear in proceedings of MFCS 2021

Motivation: Disease Spreading



Scenario: Diseases spread through physical contact.

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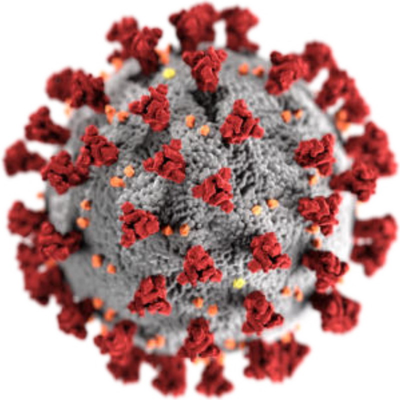

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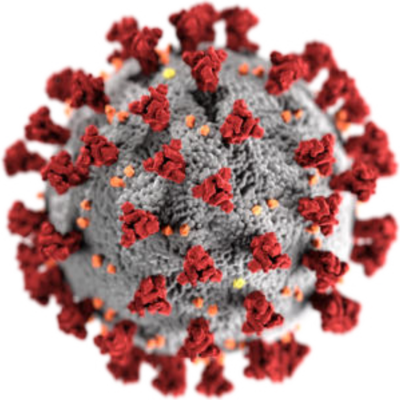

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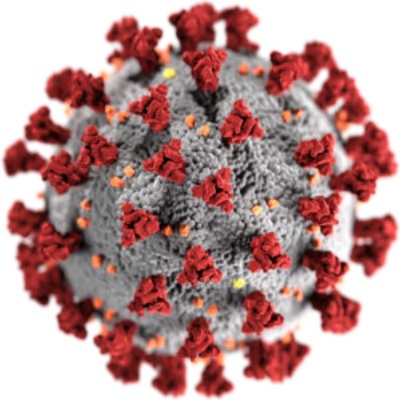

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- By **delaying** interactions.

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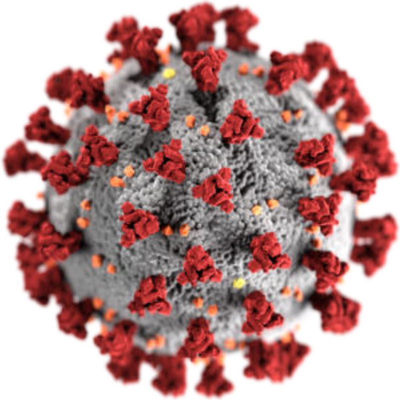

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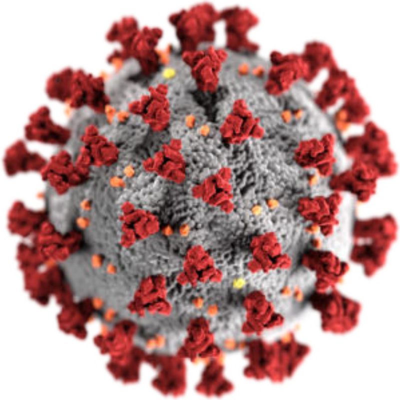

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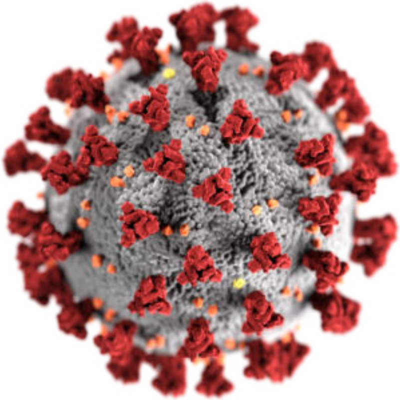

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Related Work

Minimizing Reachability by Time-Edge Modification

Modification	Known Outbreak	Unknown Outbreak

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merging	Deligkas & Potapov [AAAI '20]	?

Temporal Graphs and Temporal Reachability: Definition

Temporal Graph

A **temporal graph** $\mathcal{G} = (V, (E_i)_{i \in [\ell]})$ is a vertex set V with a list of edge sets $E_1, \ldots, E_\ell$ over V, where ℓ is the lifetime of $\mathcal{G}$.

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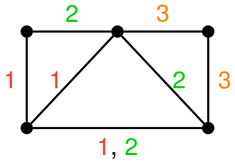

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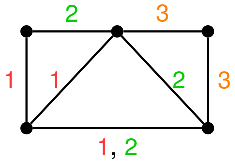

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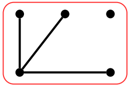

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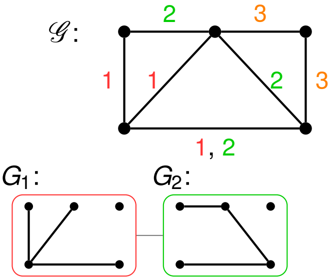

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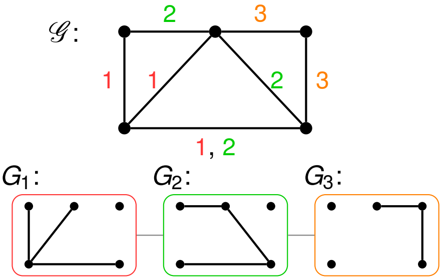

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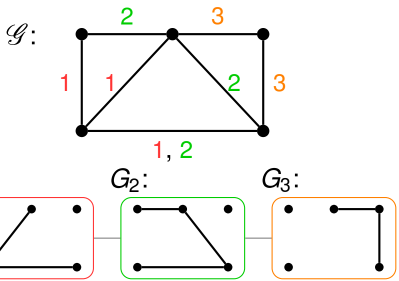

Temporal (s, z)-Path

Sequence of time edges forming a path from s to z that have:

- increasing time stamps (strict).
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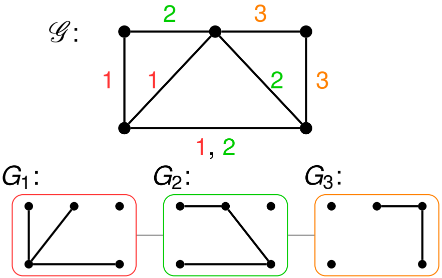

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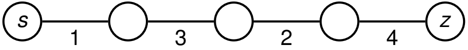

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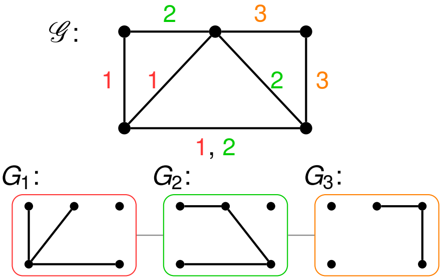

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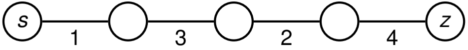

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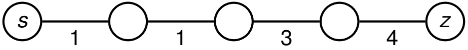

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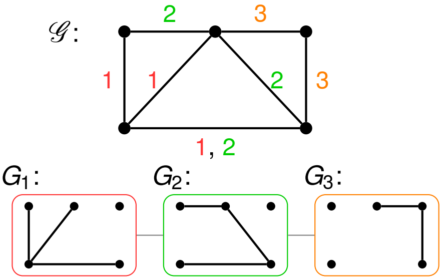

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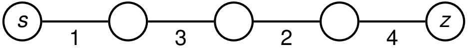

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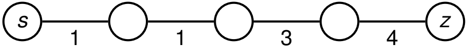

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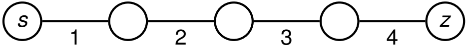

Temporal path (both strict and non-strict).

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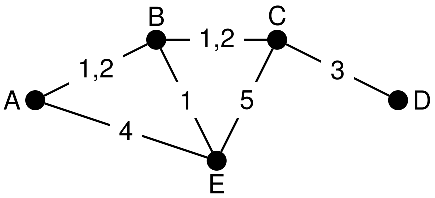

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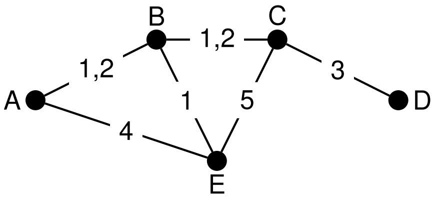

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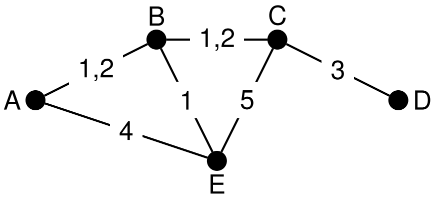

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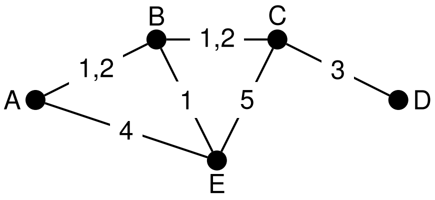

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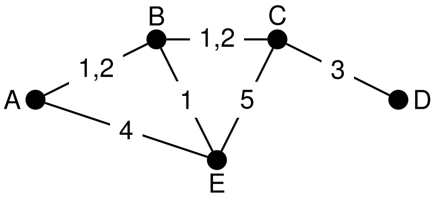

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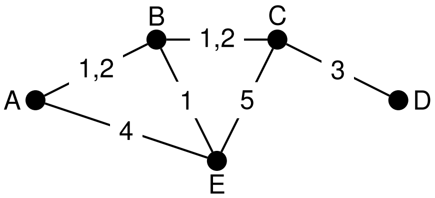

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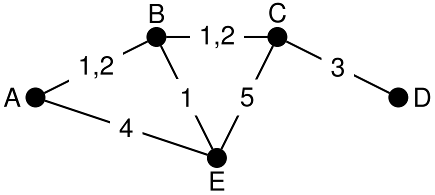

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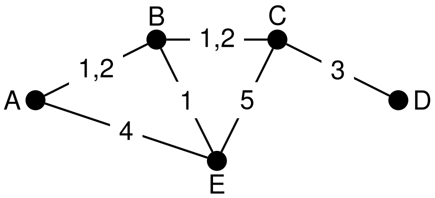

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- Fixed-parameter tractable wrt. r.

FPT Algorithm for Delaying: Idea

Algorithm Idea:

1. Move to equivalent model:

   Instead of **delaying** time-edges, **slow** them.

   $\rightsquigarrow$ On the chosen time-edges, transmission takes 2 time steps instead of 1.

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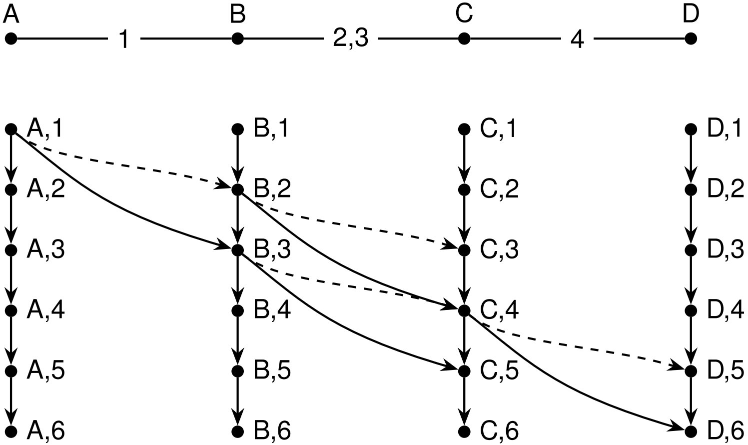

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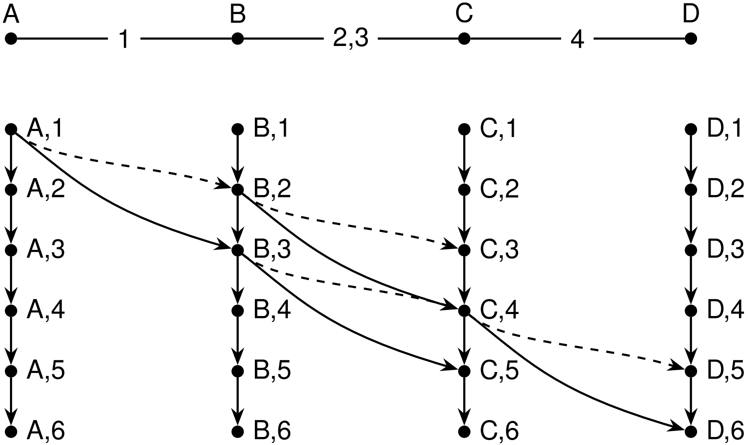

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Theorem

Minimizing Reachability by Delaying can be solved in $O(r^r \cdot k \cdot |G|)$ time.

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Arxiv link:

Thank you!