Optimizing Reachability Sets in Temporal Graphs by Delaying

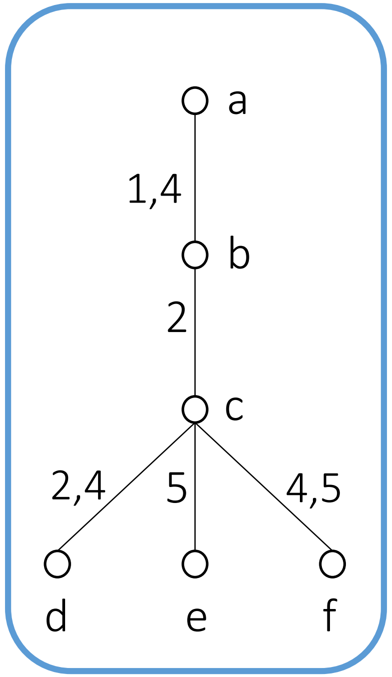
Argyrios Deligkas, Igor Potapov

Algorithmic Aspects of Temporal Graphs II

ICALP 2019

Temporal Graphs

- Graph $G = (V, E)$
- Labelling function T



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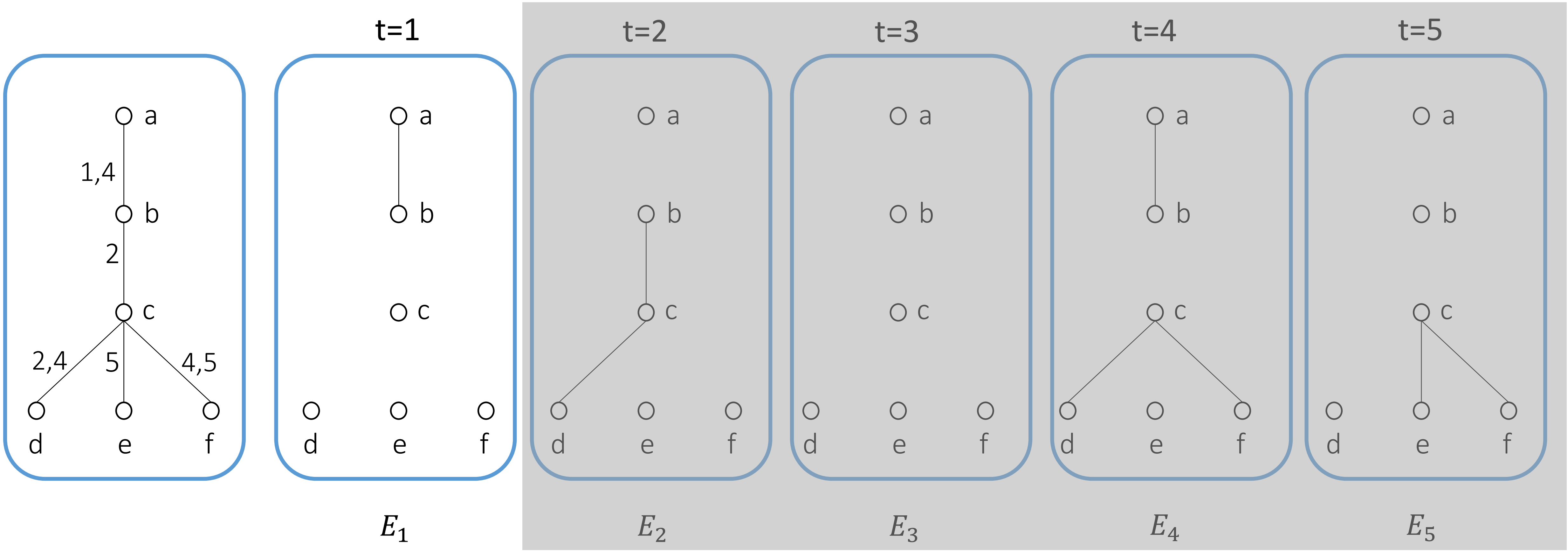

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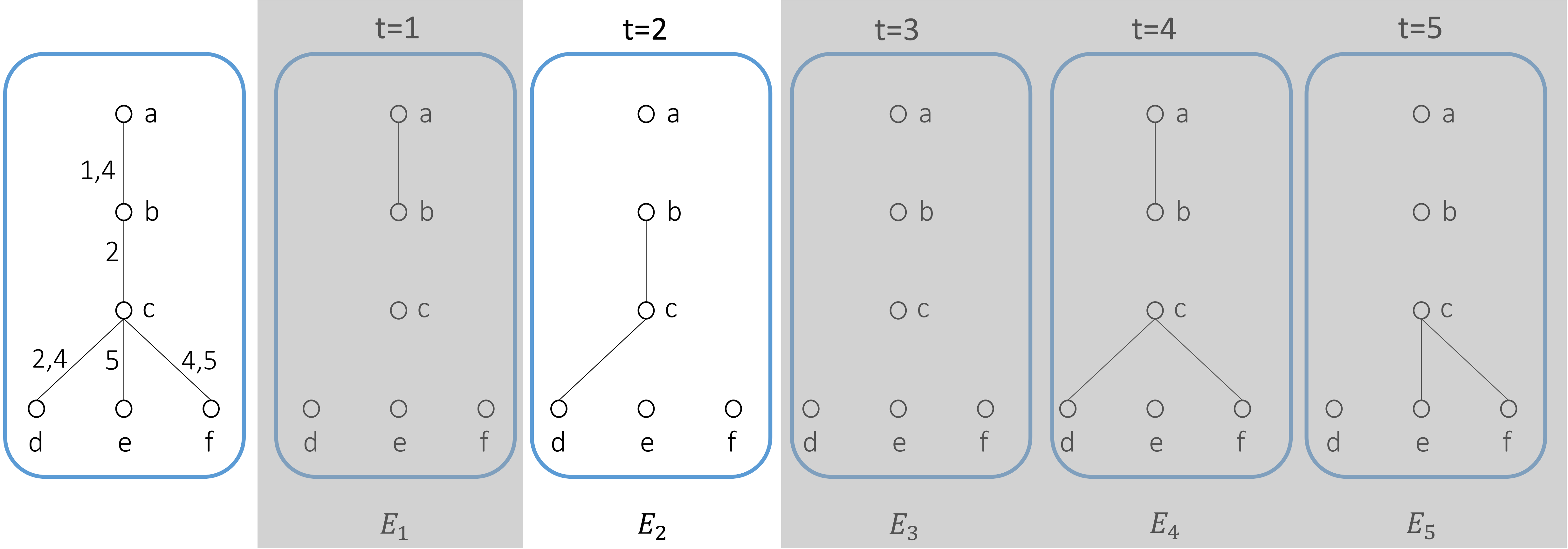

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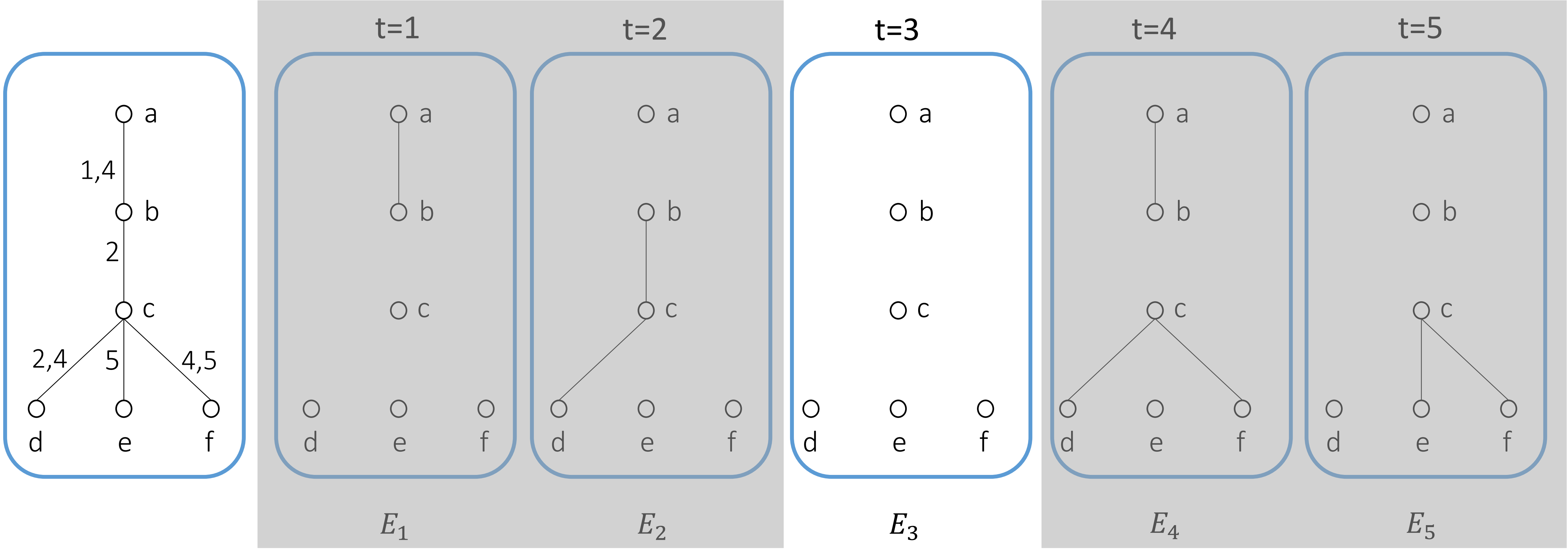

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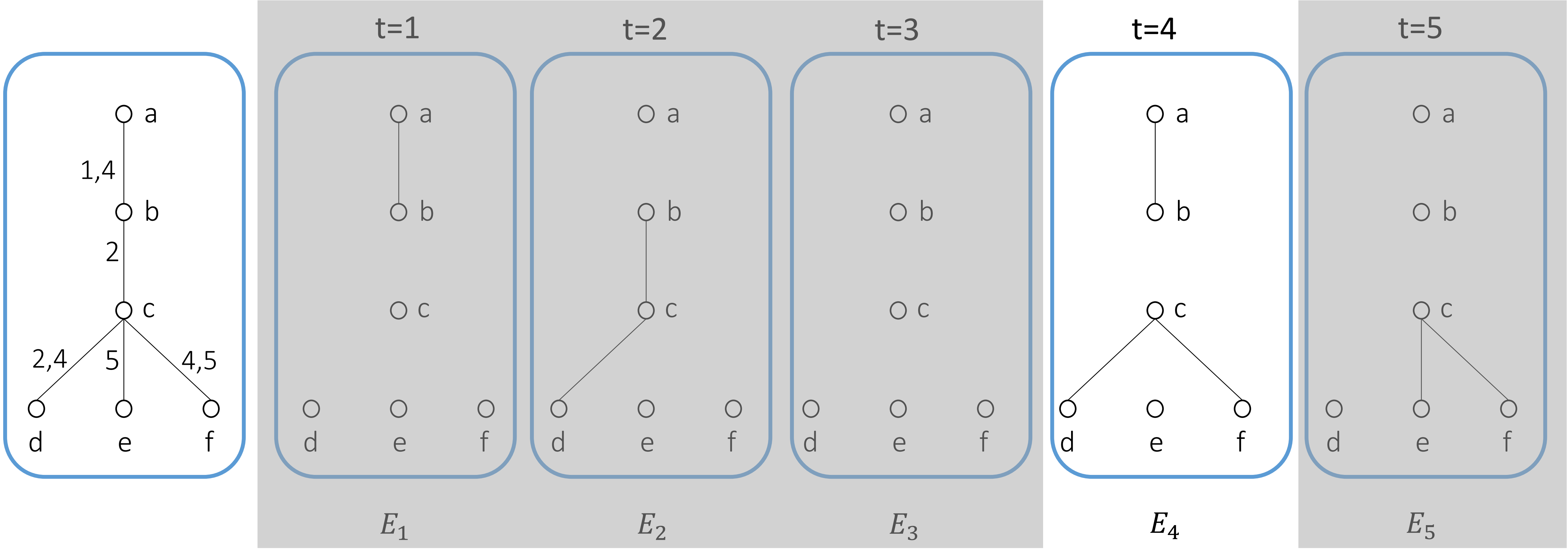

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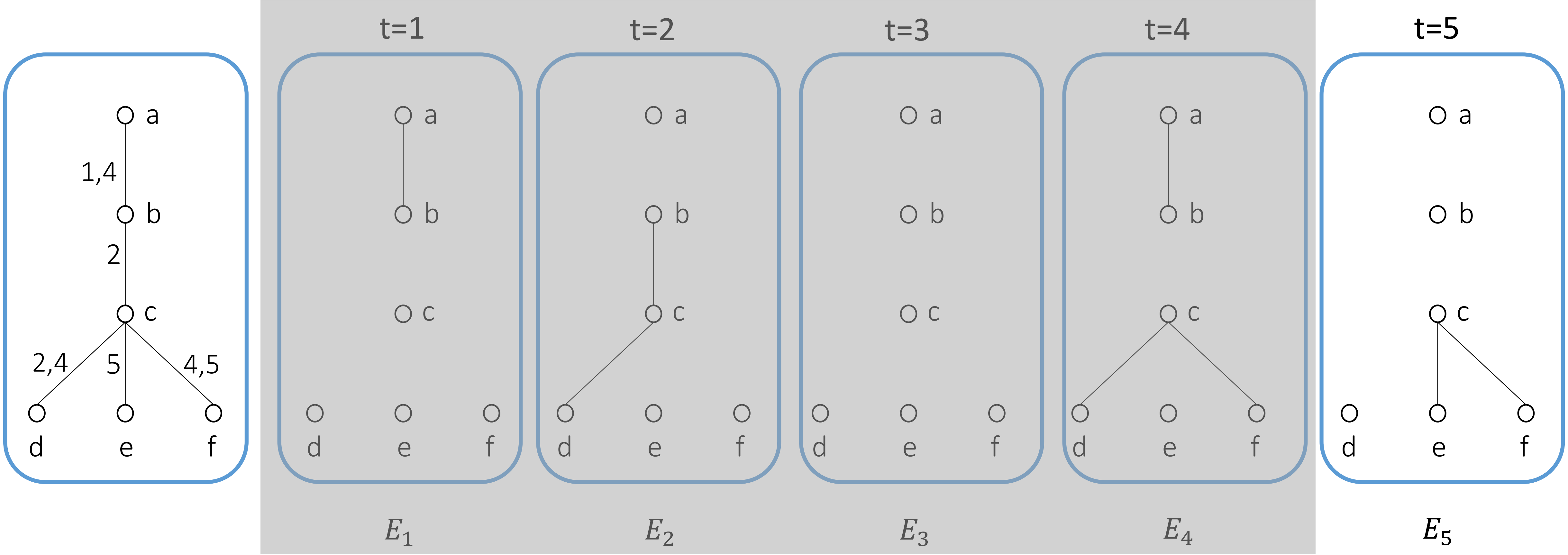

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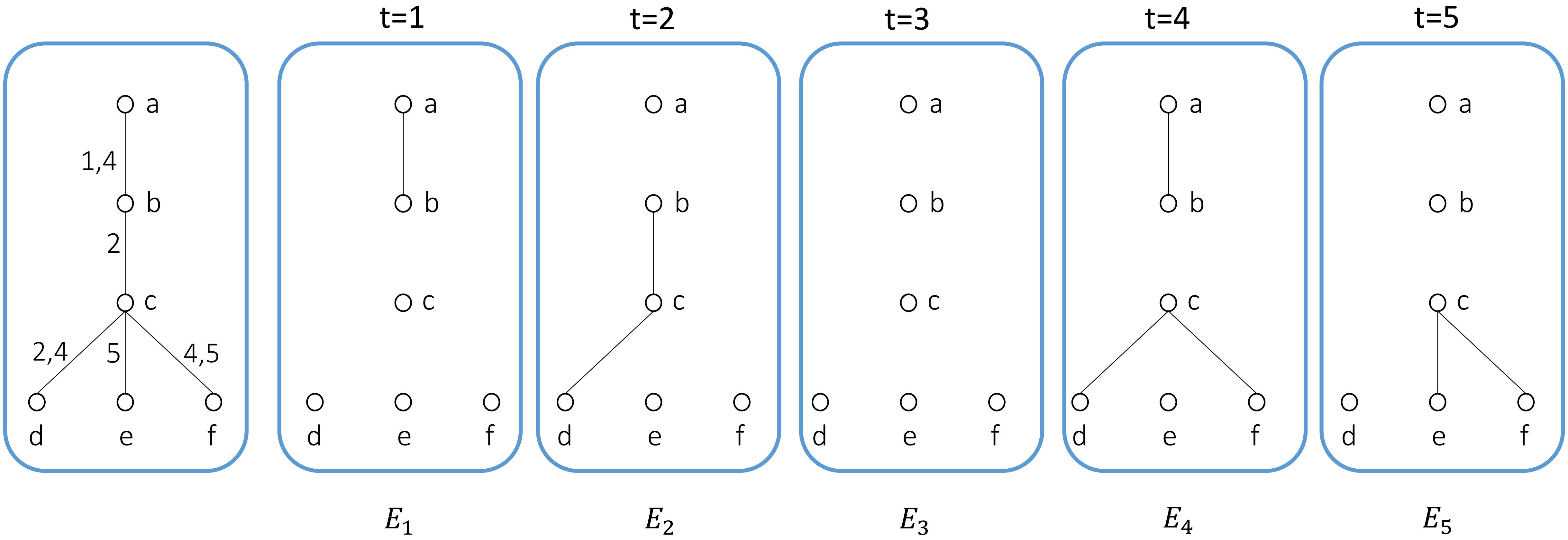

Reachability sets in temporal graphs

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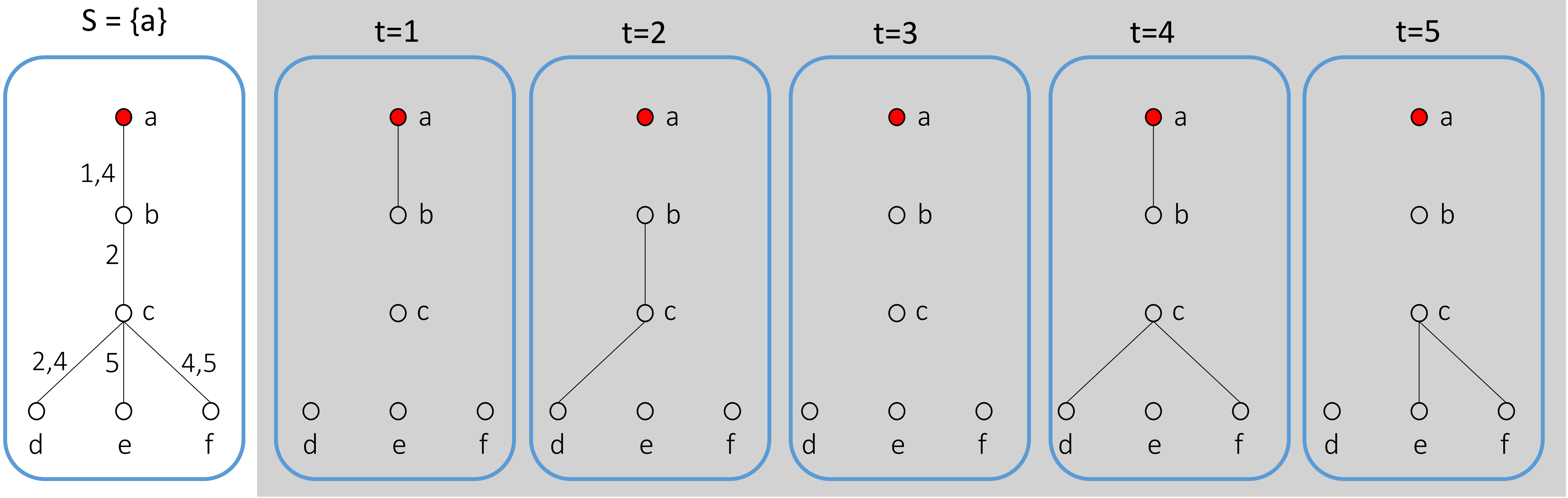

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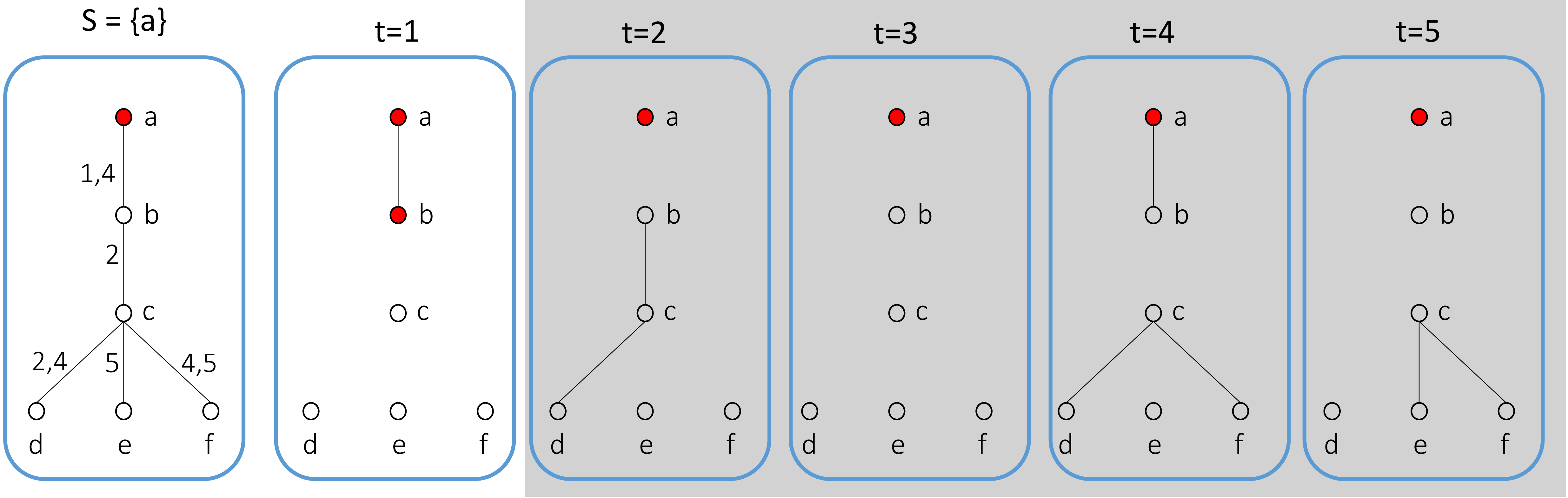

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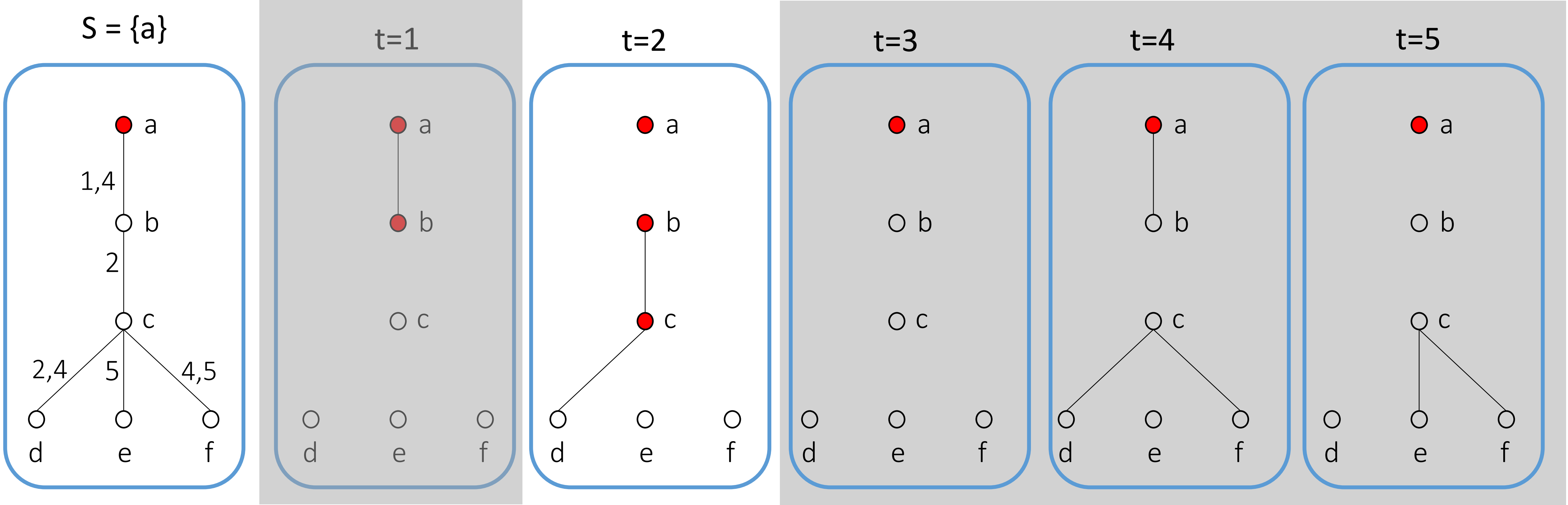

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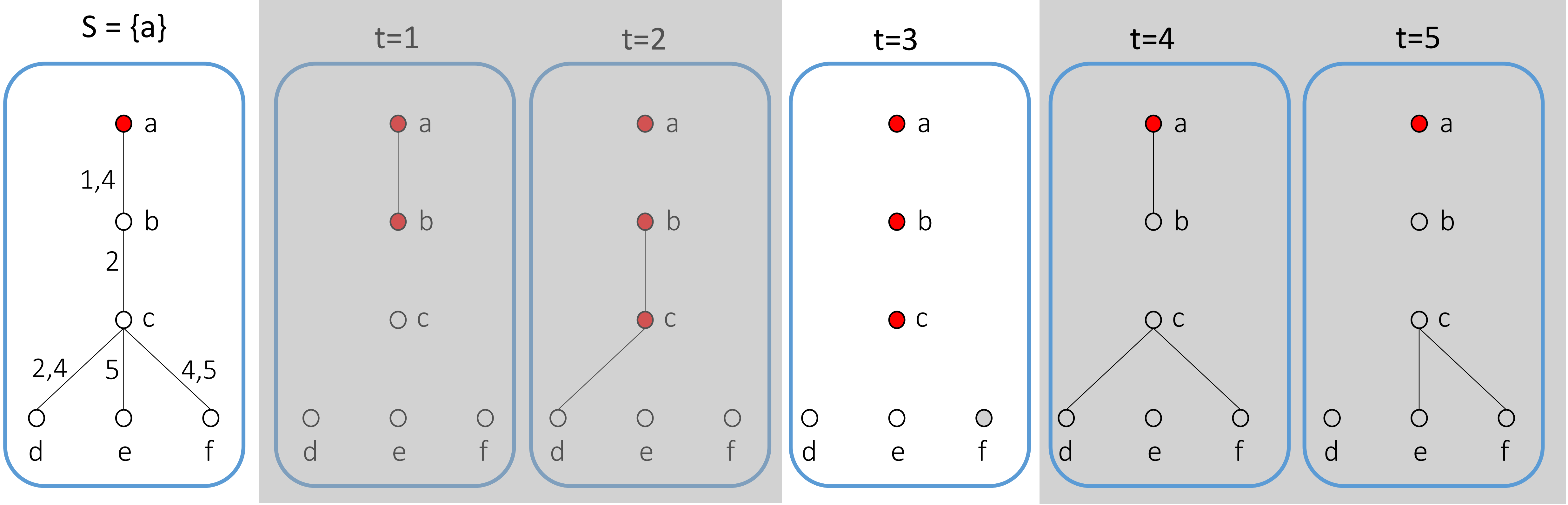

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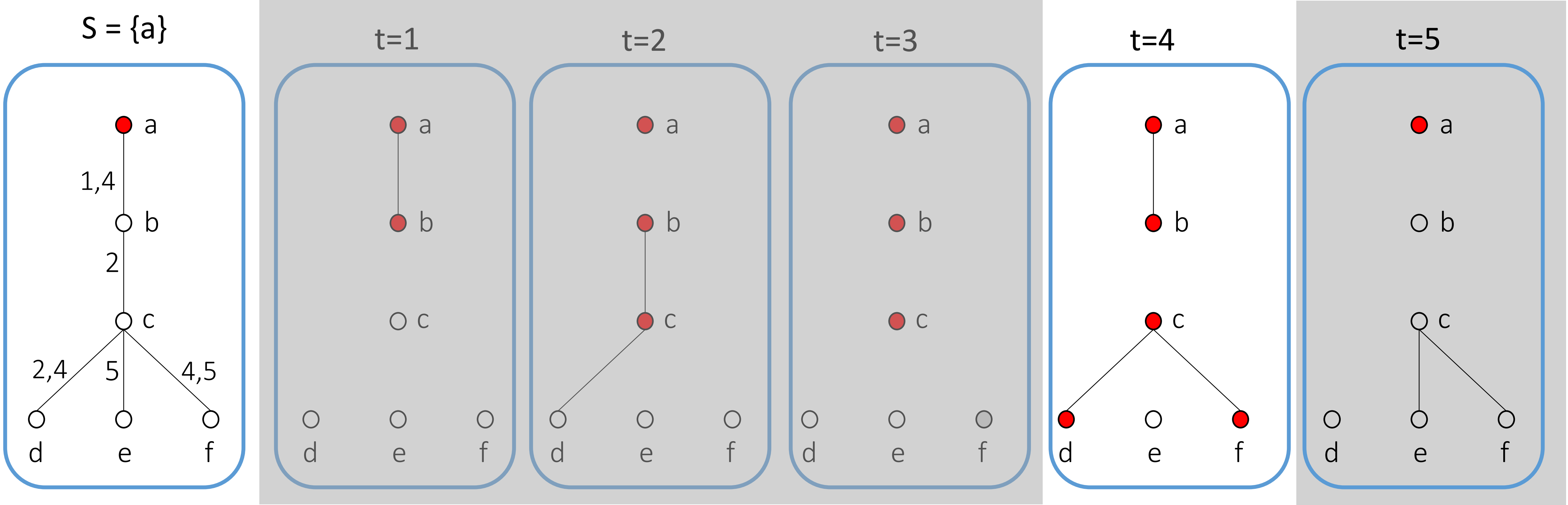

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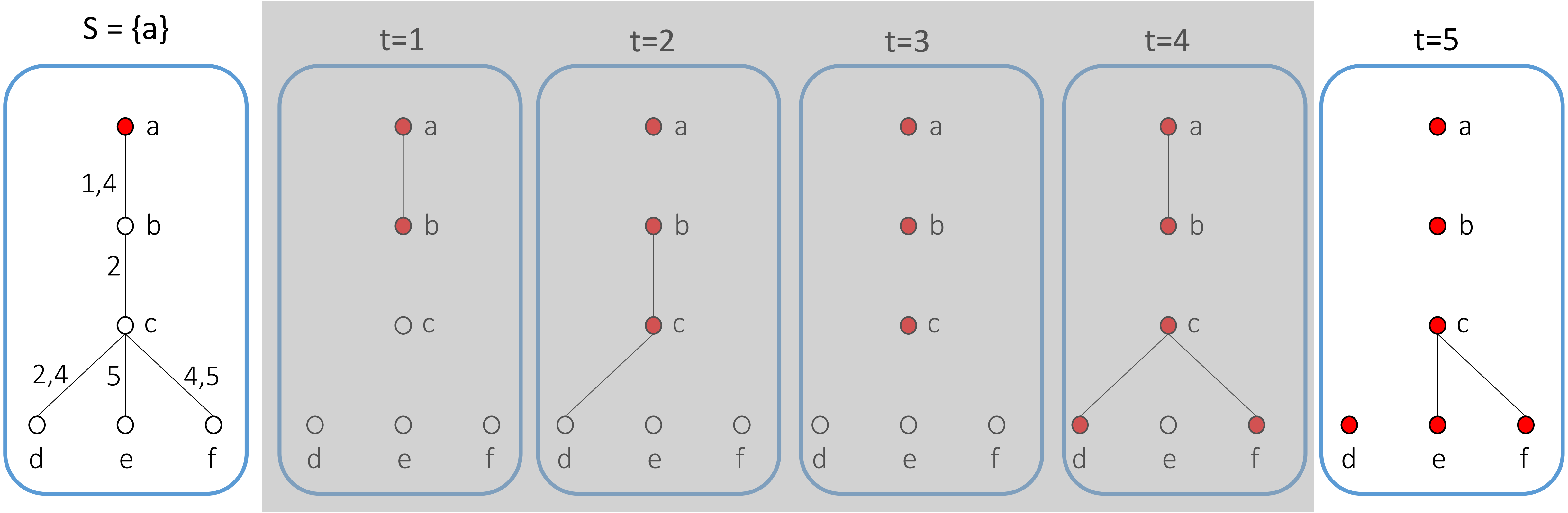

Reachability sets in temporal graphs

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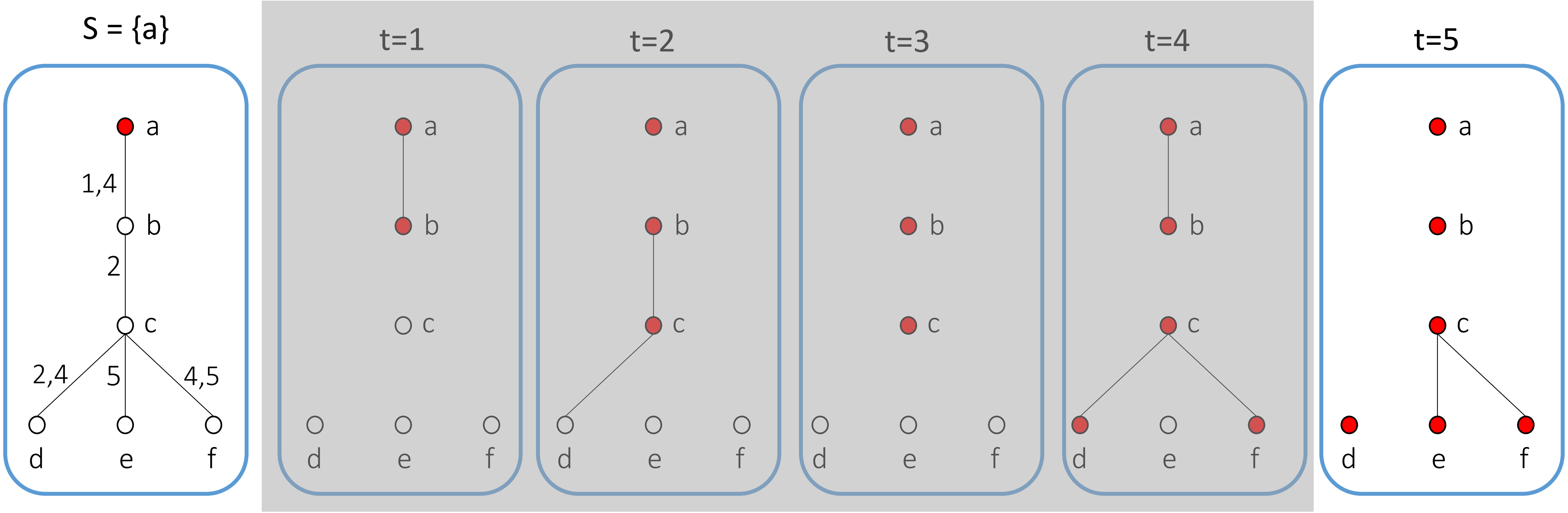
$reach(v, \langle G, T \rangle)$
Reachability set of v: Set of reachable vertices from v



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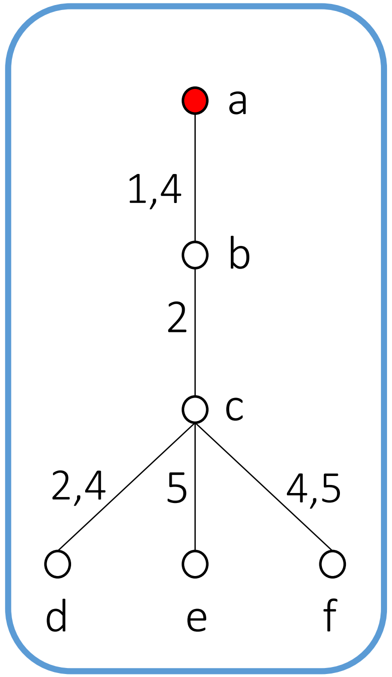
Epidemiology
Restrict spread infection

S = {a}



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How to optimize the reachability set in a temporal graph?

Epidemiology
Restrict spread infection

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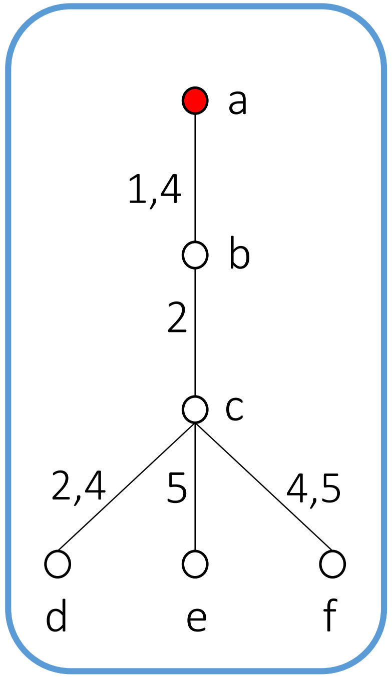

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How to optimize the reachability set in a temporal graph?

Approach 1: Reshuffling
Enright, Meeks

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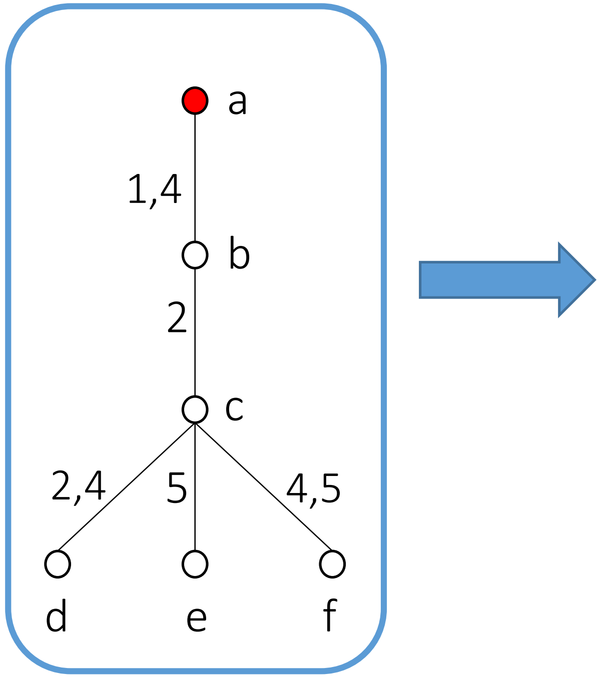

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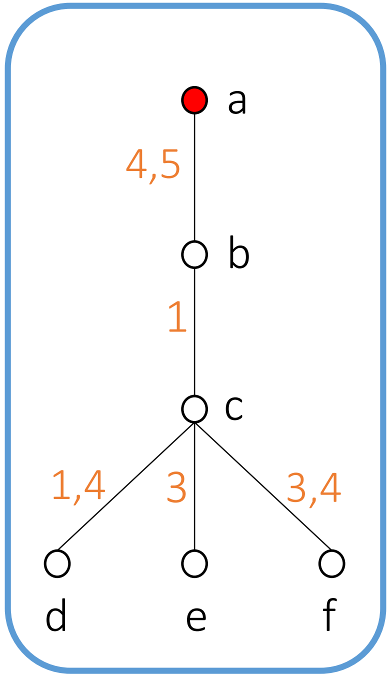

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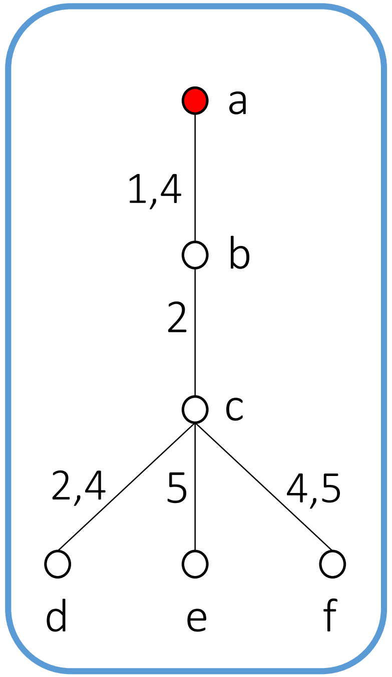

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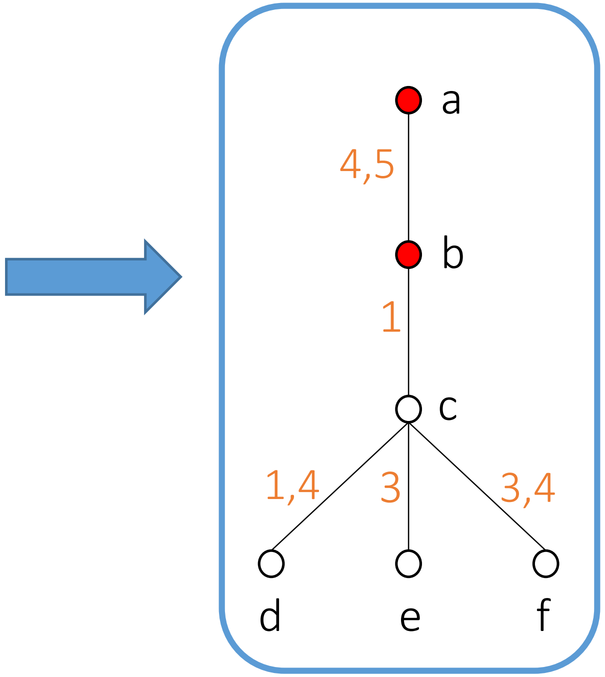

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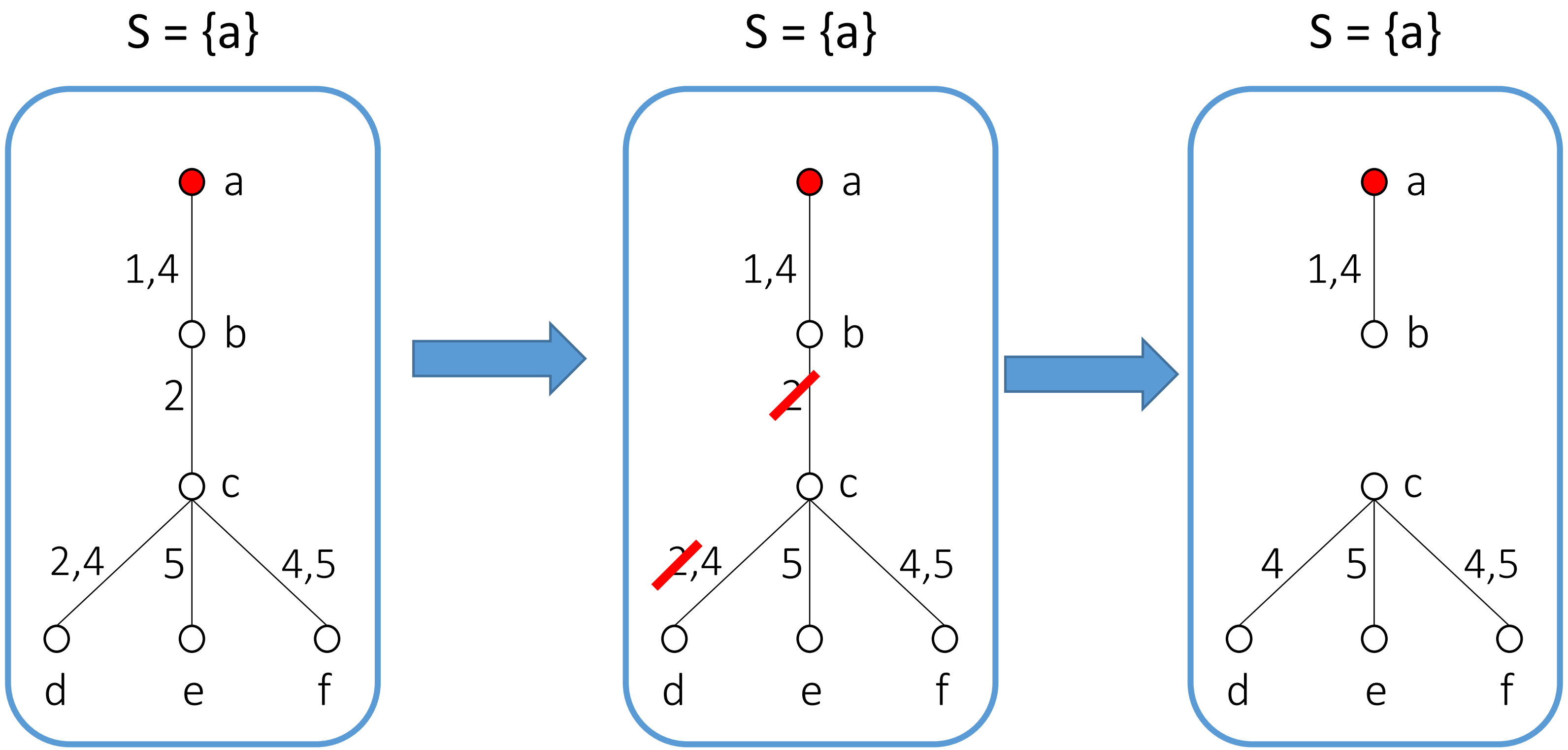
Approach 2: Edge deletion
Enright, Meeks, Mertzios, Zamaraev



Delete all edges with label 2

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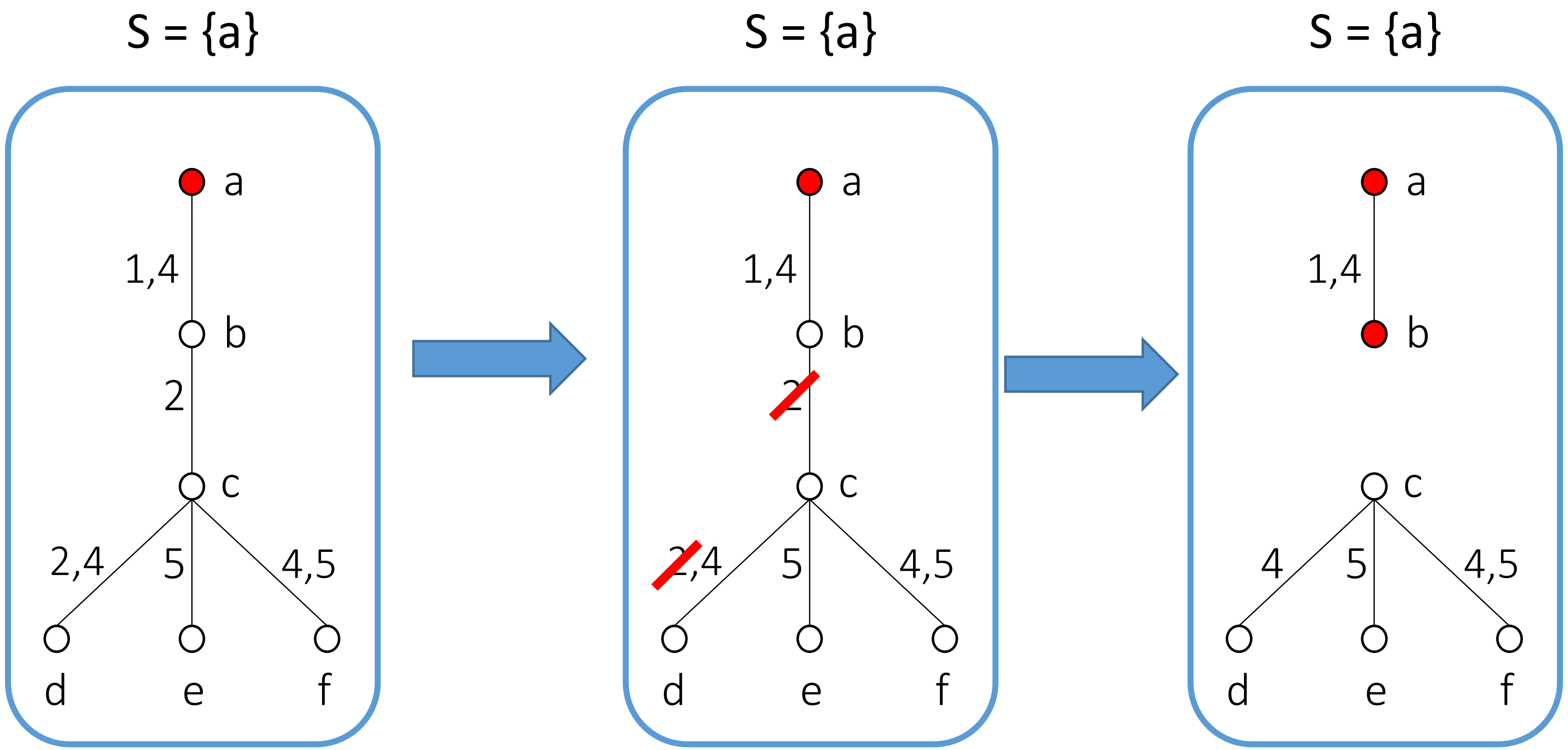

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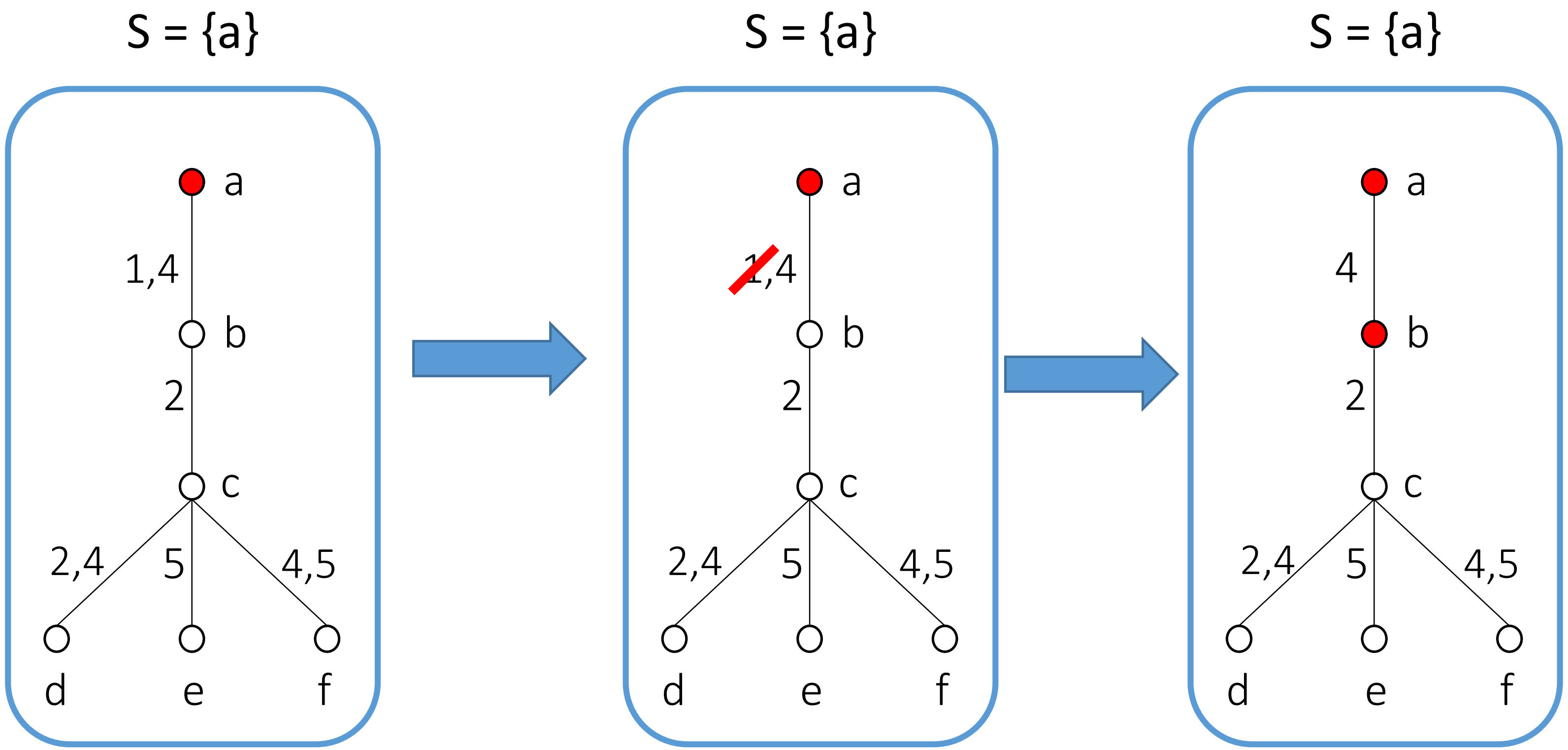

Delete label 1 from edge ab

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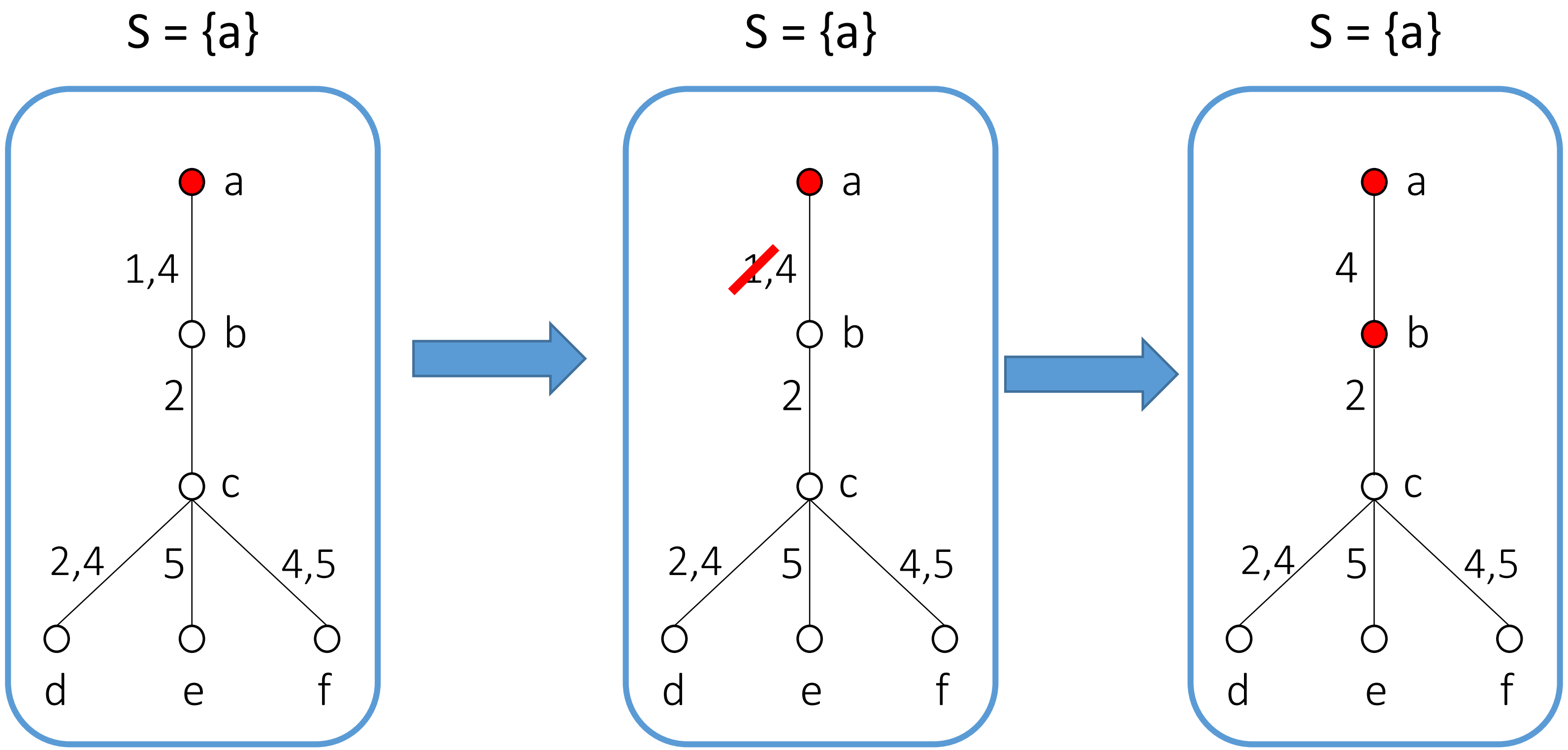

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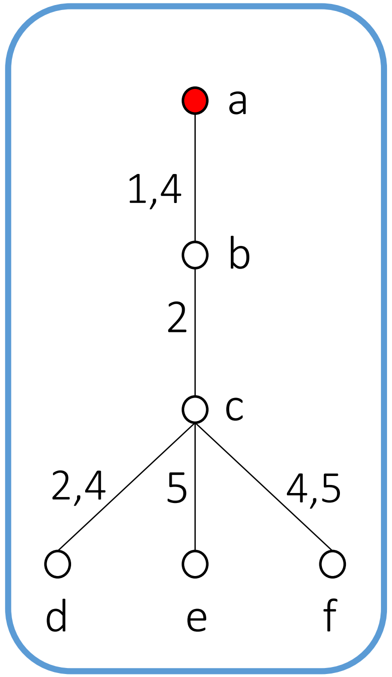

How to optimize the reachability set in a temporal graph?

Approach 1: Reshuffling
Enright, Meeks

Not always possible in real-life networks
(too many changes)

Approach 2: Edge deletion
Enright, Meeks, Mertzios, Zamaraev

Can create deadends in the network
(blocks the flow)

Our approach: Delaying

- Graph $G = (V, E)$
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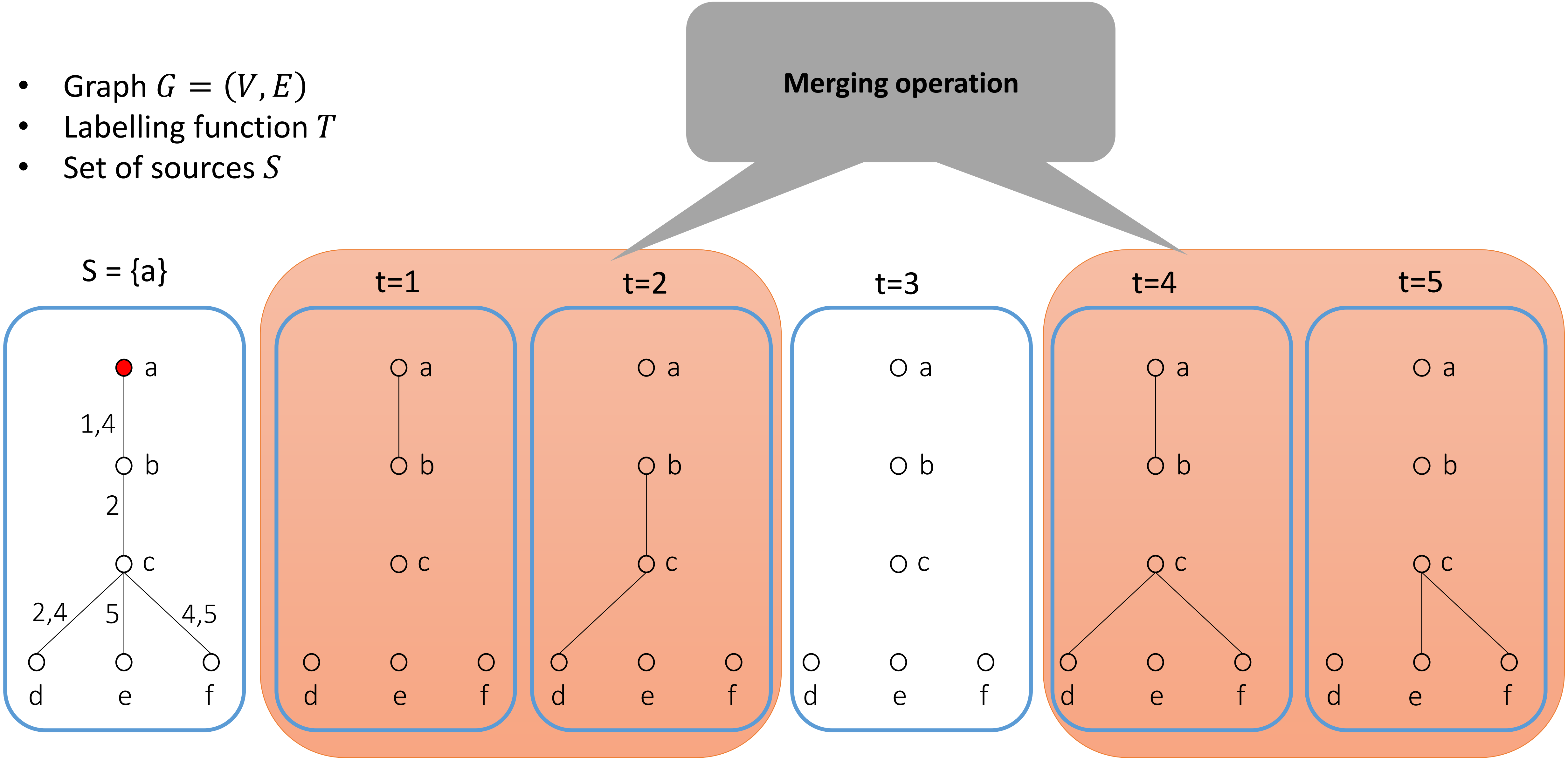

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- Graph $G = (V, E)$
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λ-merge: $E_i, \ldots, E_{i+\lambda-1}$

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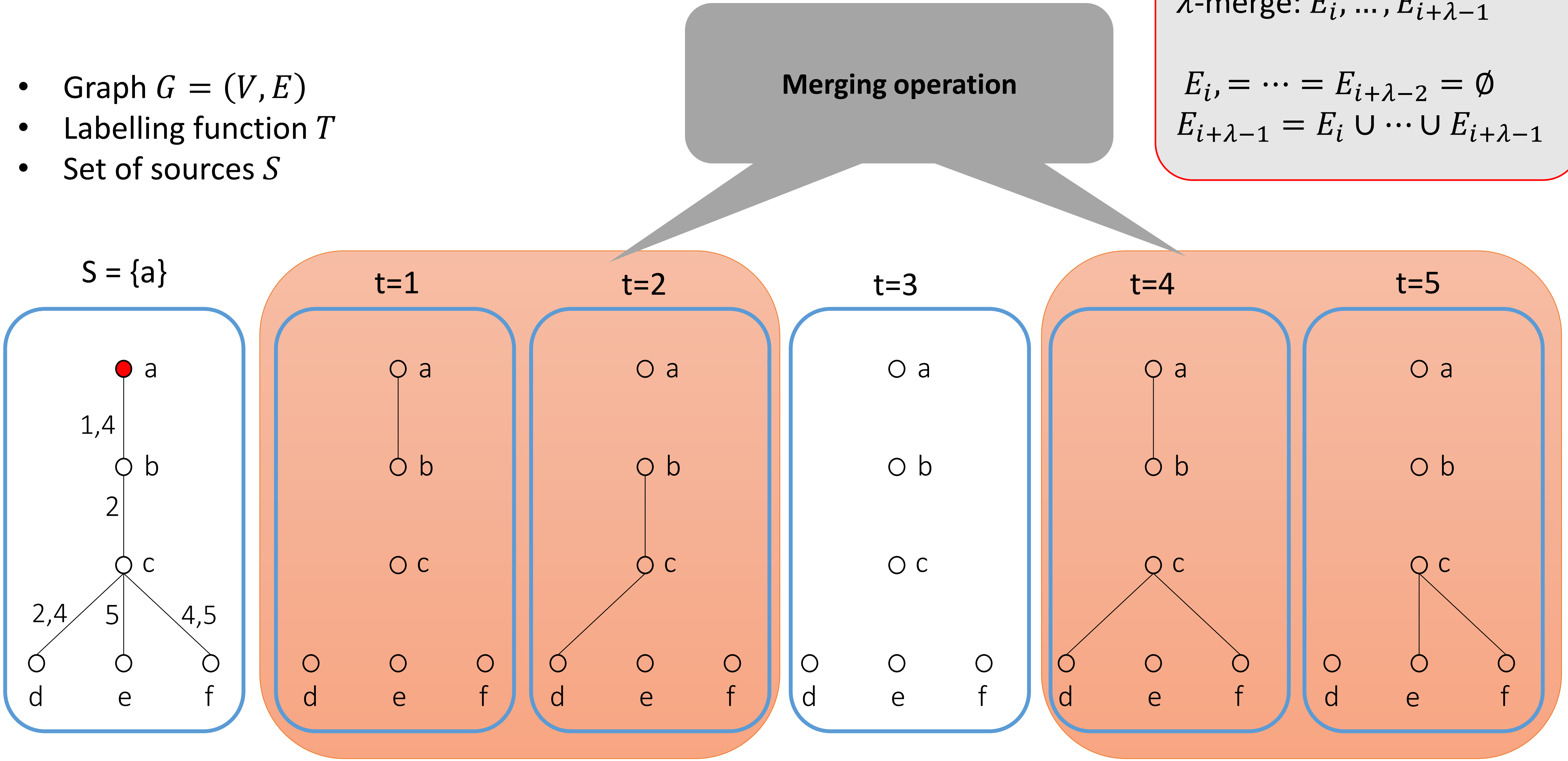
$E_{i+\lambda-1} = E_i \cup \cdots \cup E_{i+\lambda-1}$



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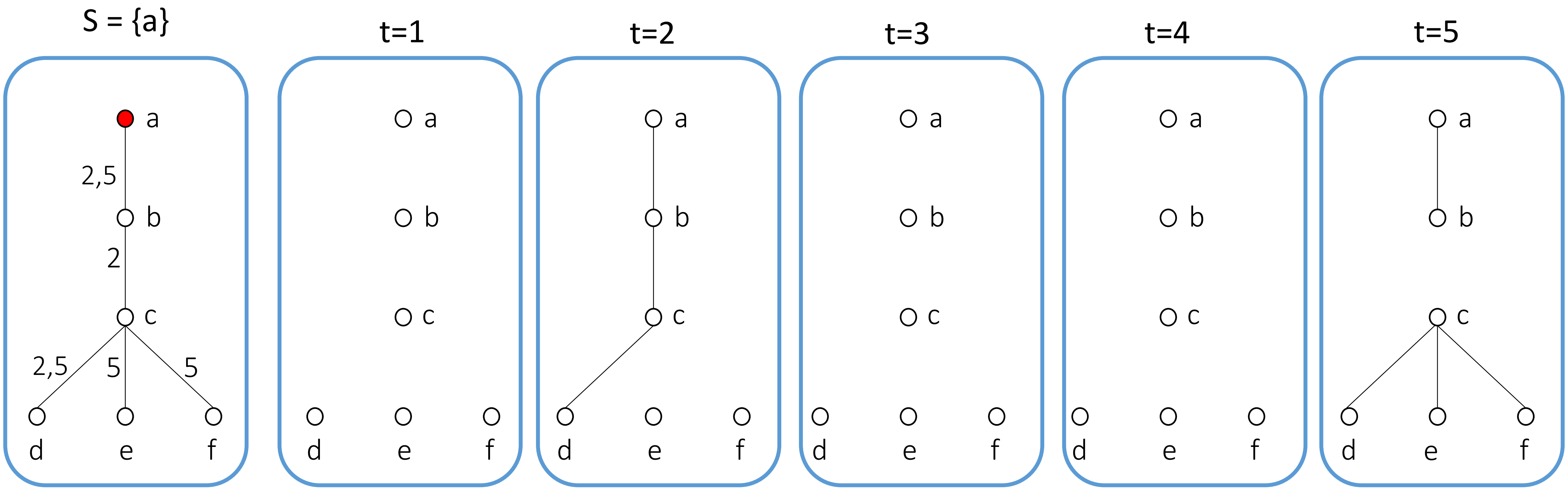

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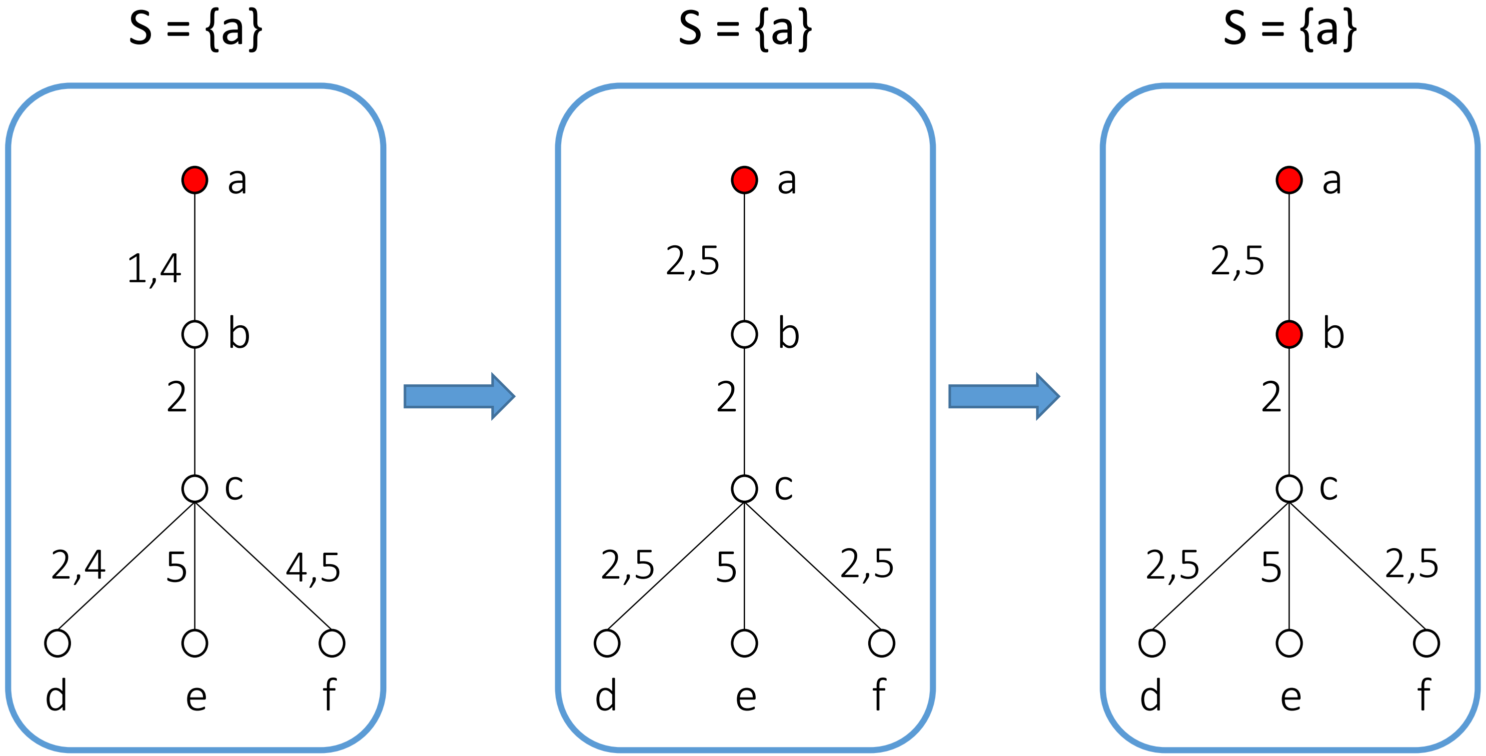

- Minimum modification/disturbance of the original network
- Does not create deadends
- Intuitive

Merging Schemes

A (λ, μ)-merging scheme uses at most/least μ **independent** λ-merges

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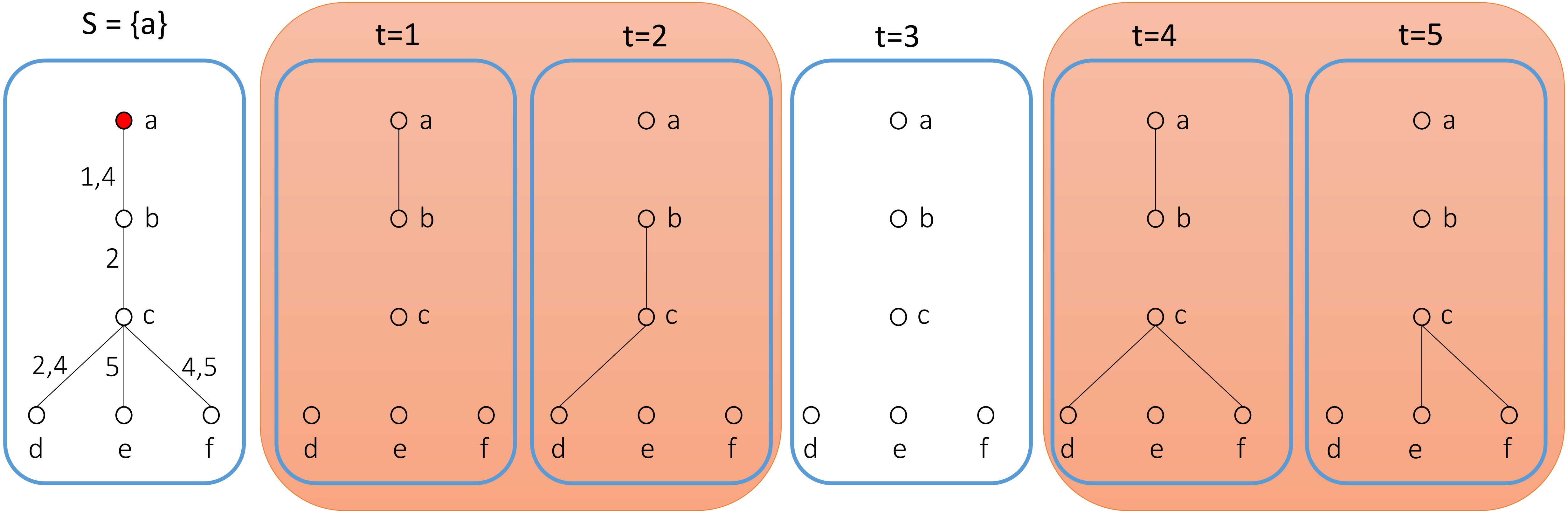

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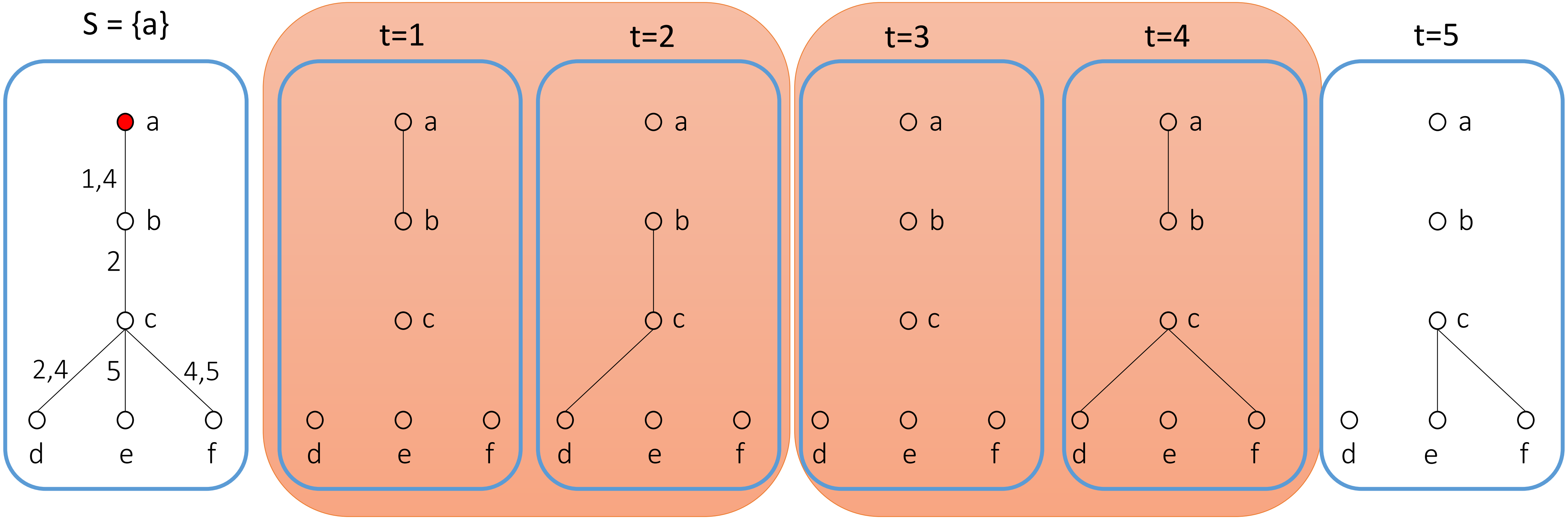

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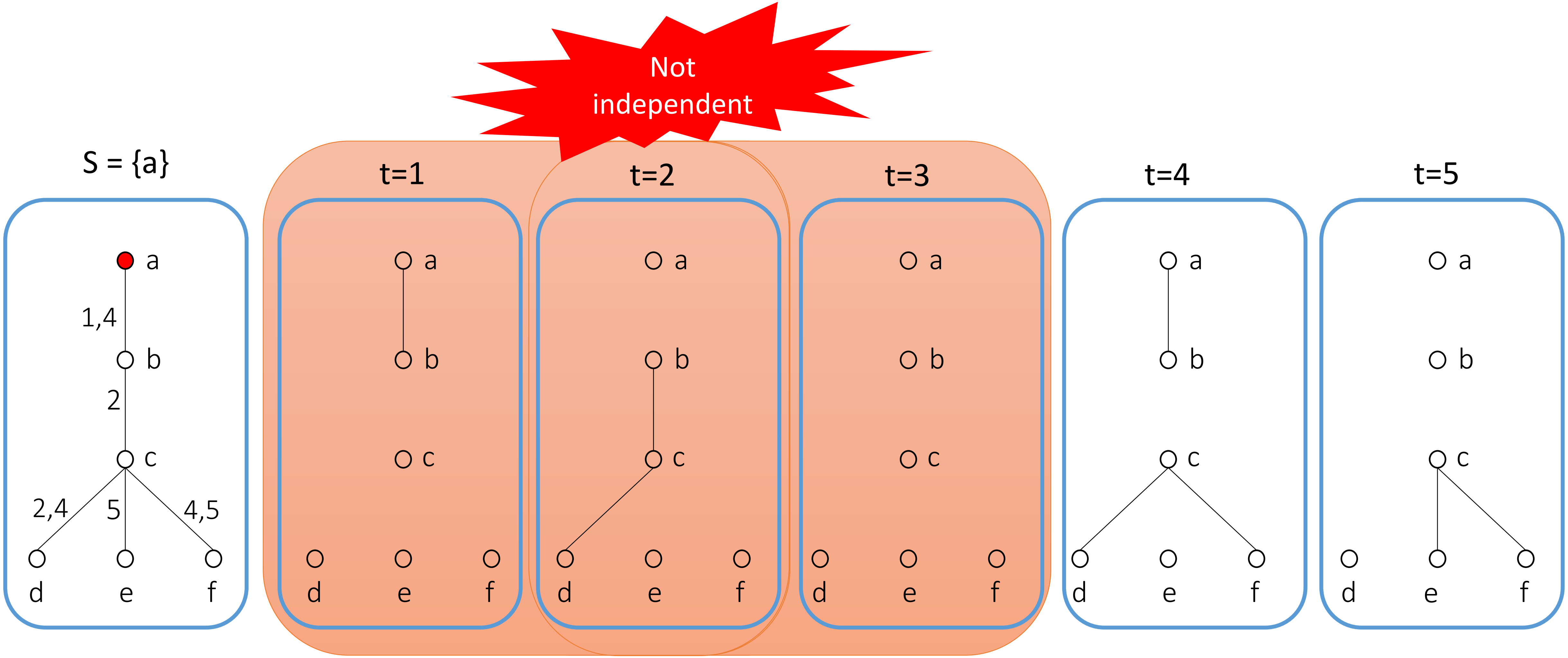

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(λ, μ)-merging scheme uses **at most** μ independent λ-merges

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Our Results

Problem	Graph	# Sources	# Labels/Edge	# Edges/Step
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MinReach MinMaxReach MinAvgReach	Tree max degree 3	1	1	1
MaxReach	Path	O(n)	1	4
MaxReach MaxMinReach MaxAvgReach	Bipartite Max degree 3	1	1	4
MaxReach MaxMinReach MaxAvgReach	Tree max degree 3	1	1	10

NP-hard for every λ

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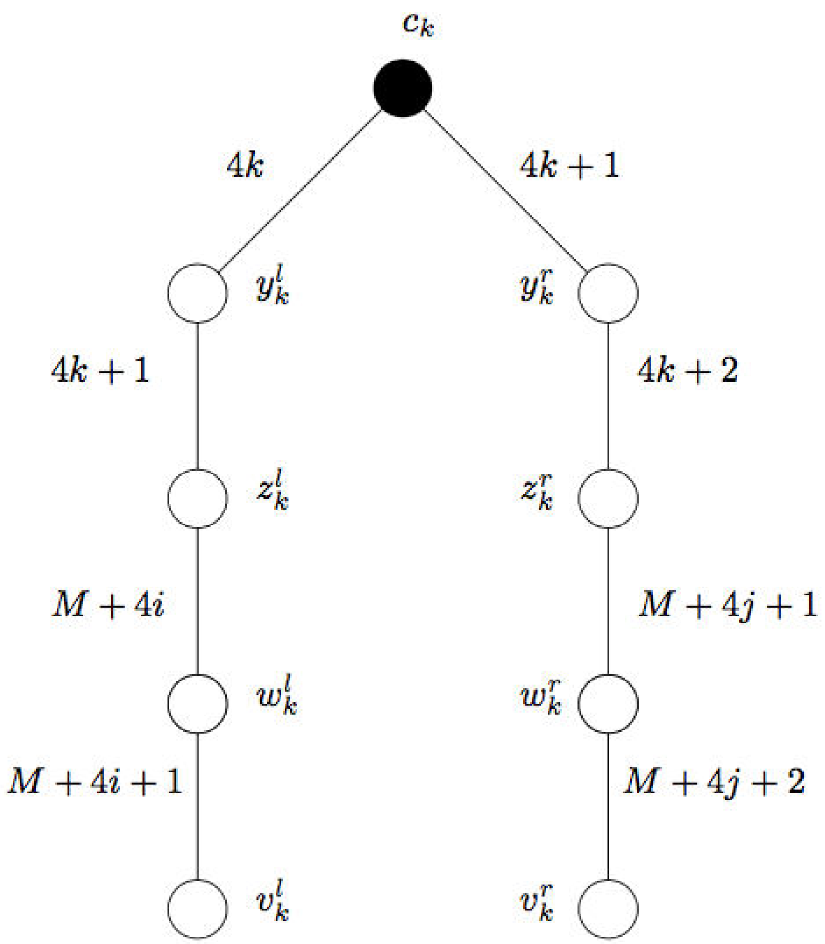
NP-hard for every **λ**

- **DAGs**
- **Unit disk graphs**
- **Approximation preserving, no PTAS**

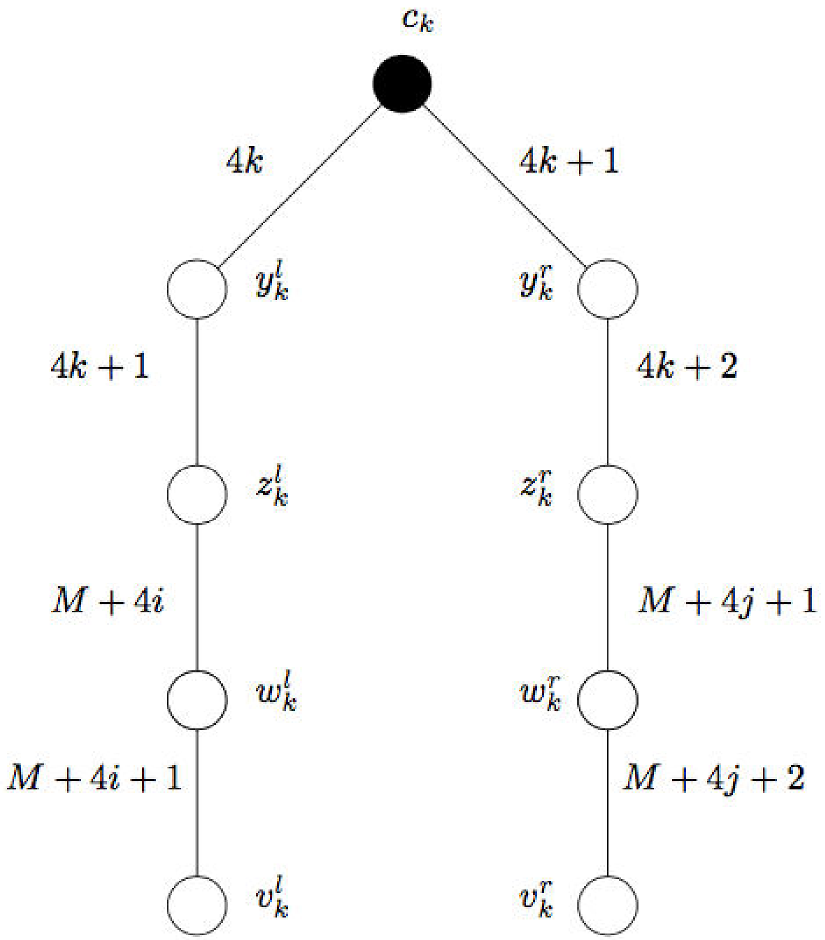
Idea



Reduction from Max2SAT(3)

Open Questions

- Approximation Algorithms
- Tractable Cases/Graph Classes
- FPT algorithms



Thanks!!