Computing Parameters of Sequence-based Dynamic Graphs

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**This is a joint work with Arnaud Casteigts, Yessin M. Neggaz, and Joseph G. Peters.

Dynamic Networks

- Highly dynamic networks?

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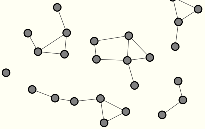
- How changes are perceived?
  - ~~Faults and Failures?~~
  - Nature of the system
  - Change is normal



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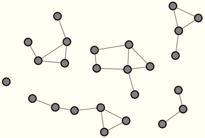

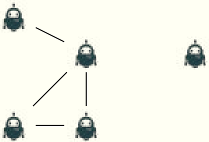

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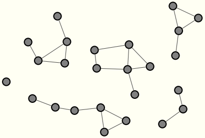

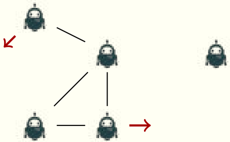

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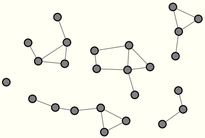

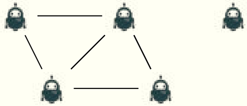

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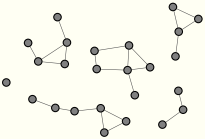

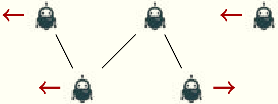

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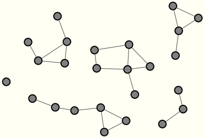

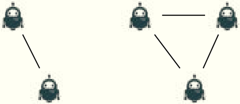

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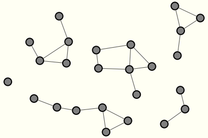

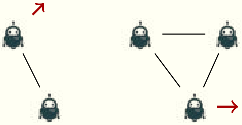

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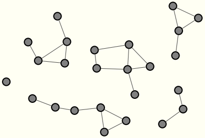

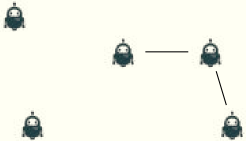

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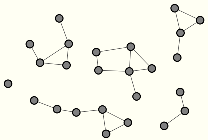

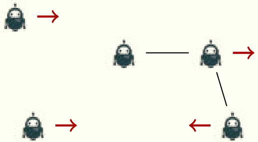

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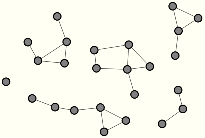
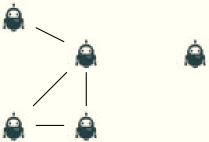
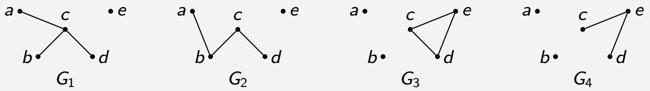




Dynamic Graphs

Dynamic graphs: Various forms: TVG, Evolving graphs



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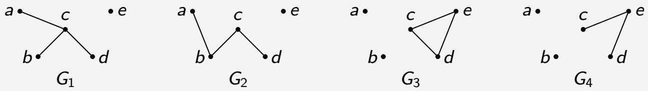

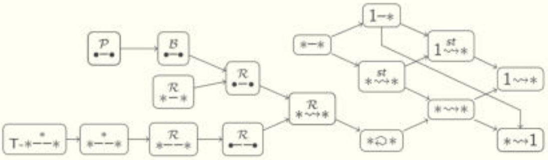

Dynamic graphs classes: [Casteigts, Flocchini, Quattrociocchi et Santoro, 2011]

Temporal Connectivity

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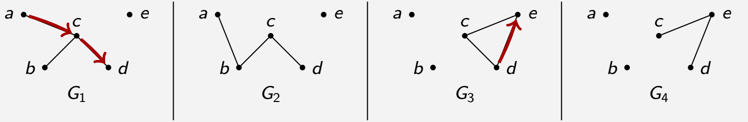
$\mathcal{G} = (V, E_i)$



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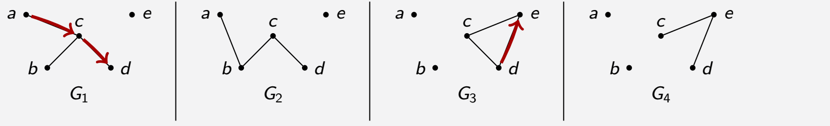

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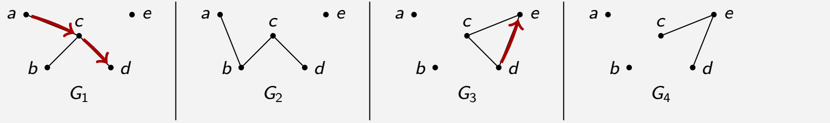
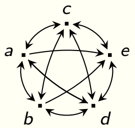


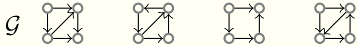
- **Temporal connectivity** $\iff \forall u, v \in V, u \rightsquigarrow v$.
- **Transitive closure** of the journeys: reachability over time *[Bhadra and Ferreira, 2003]*

$\mathcal{G}^*$



- $\mathcal{G}$ is **temporally connected** $\Leftrightarrow$ Transitive closure $\mathcal{G}^*$ is **complete**

High-level Strategy



High-level Strategy

$G_{(1,4)}$

- High-level strategies that work directly at the graph level
- Elementary graph-level operations

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Finding the *temporal diameter* of a given dynamic graph $\mathcal{G}$, i.e. the smallest duration in which there exists a journey from any node to all other nodes.

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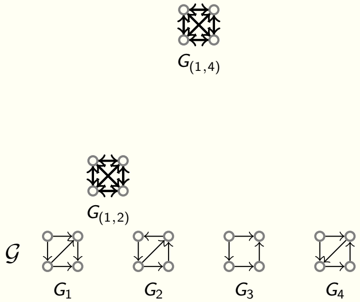

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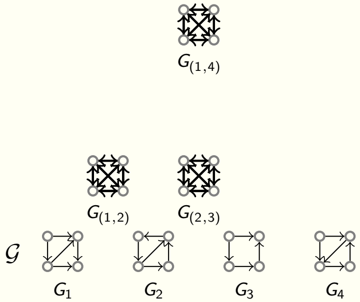

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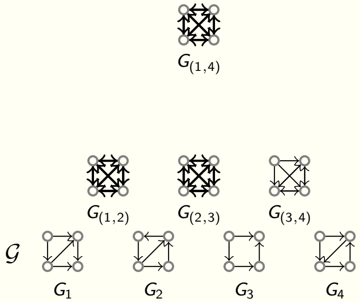

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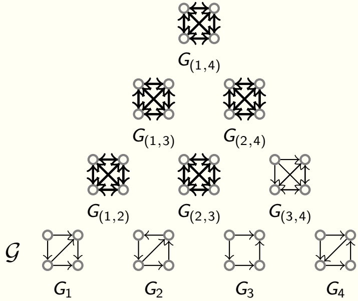

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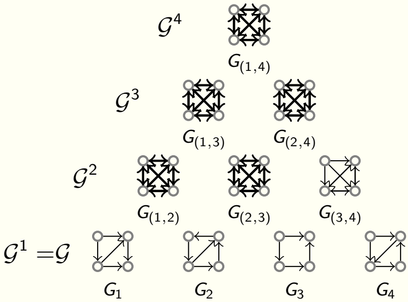

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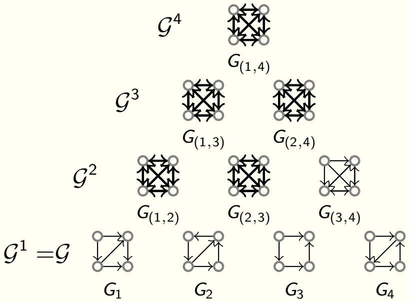

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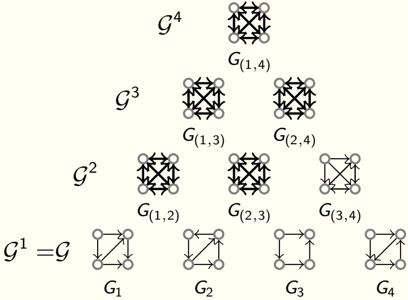
$$\Longleftrightarrow$$

Finding the smallest d such that every super node in row $\mathcal{G}^d$ is a complete graph (i.e. every subsequence of length d is temporally connected).

High-level Strategy

- Transitive closures
- Completeness test
- Transitive closures concatenation



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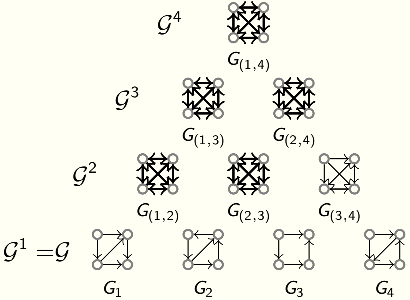

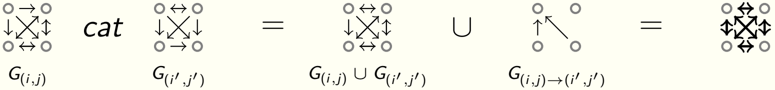

Temporal Diameter Computation

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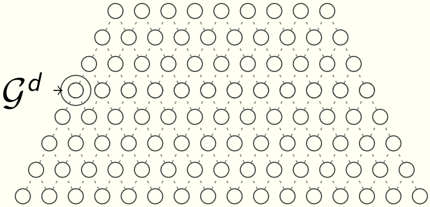
Decision version (given d)



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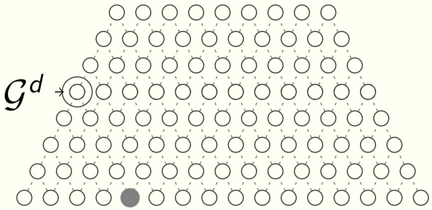

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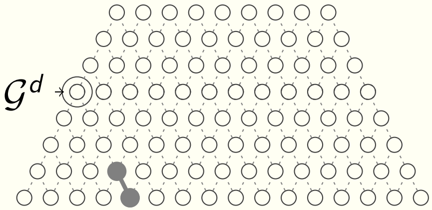

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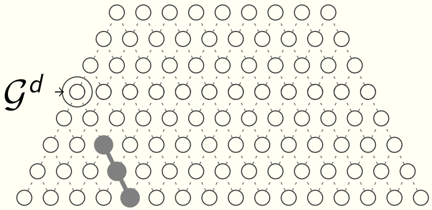

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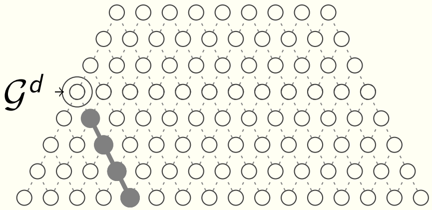

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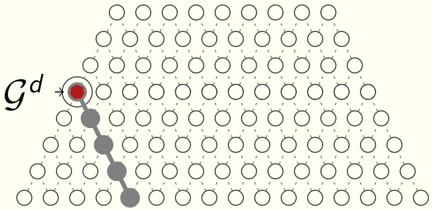

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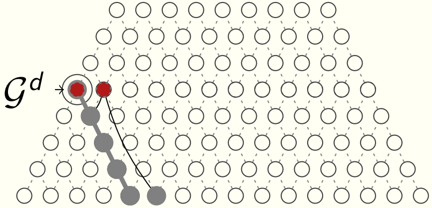

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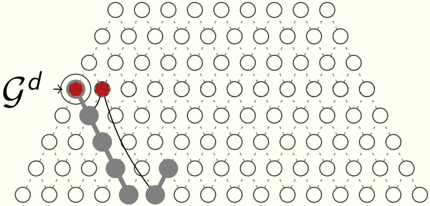

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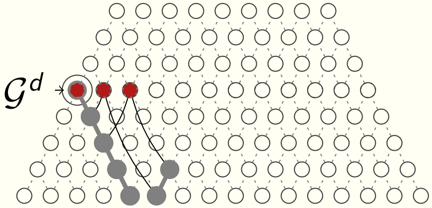

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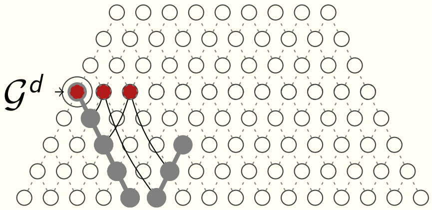

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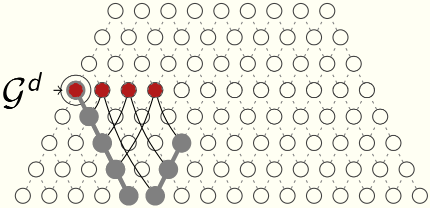

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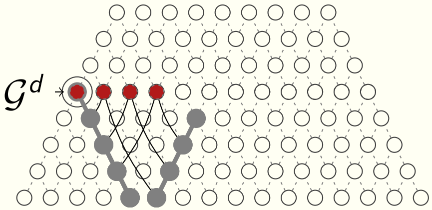

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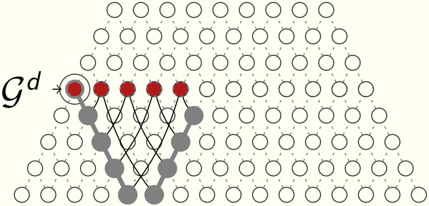

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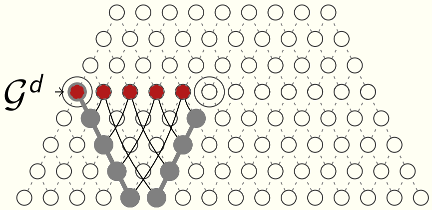

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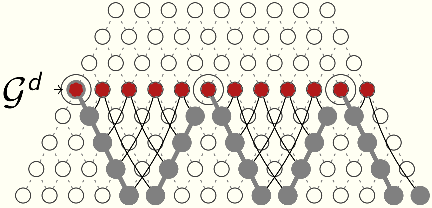

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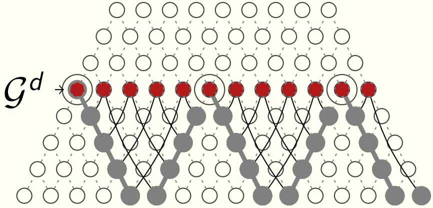

$O(\delta)$ **elementary operations per row**

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Minimization version (find the temporal diameter d)

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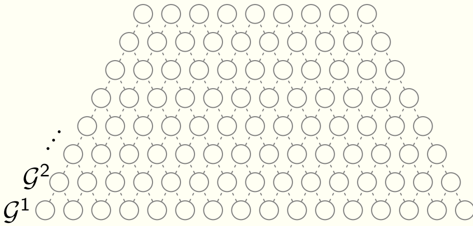

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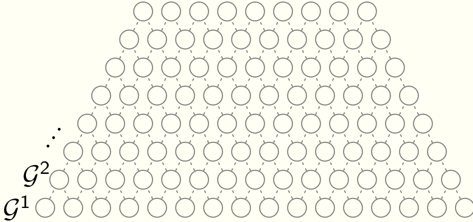

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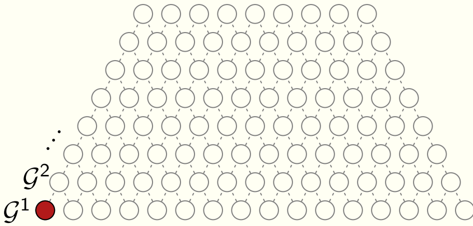

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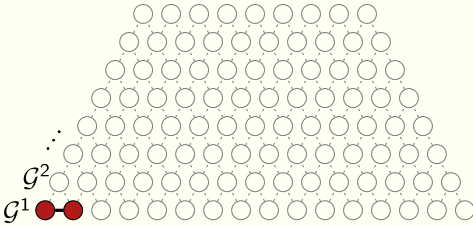

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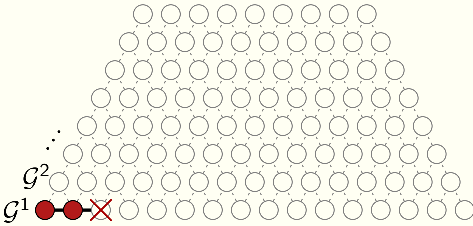

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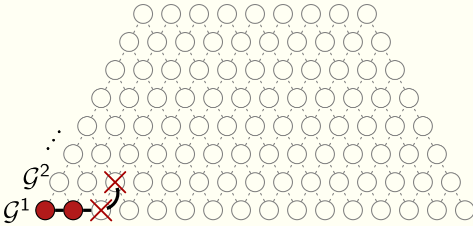

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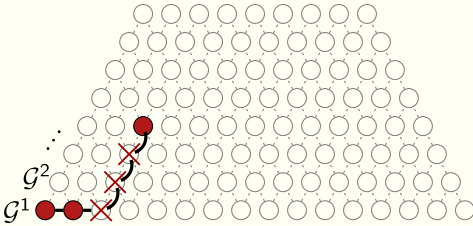

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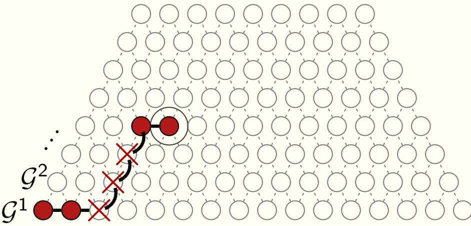

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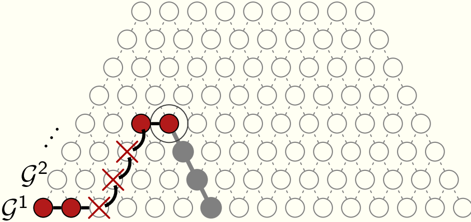

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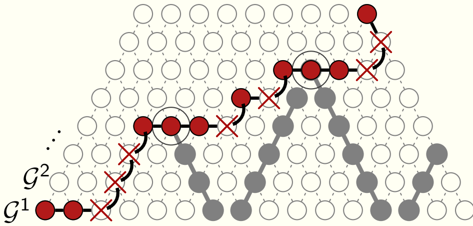

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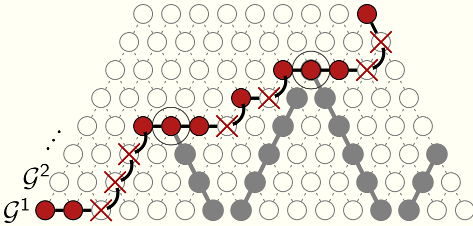
- Strategy: ascending walk
- The total length of the ladders is $O(\delta)$
- At most $O(\delta)$ binary concatenation and completeness tests



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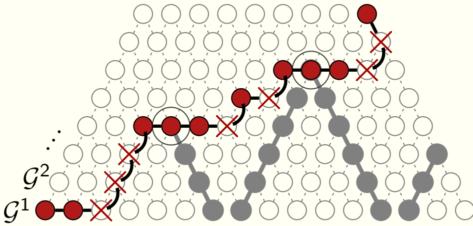

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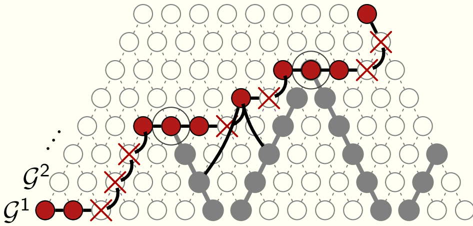

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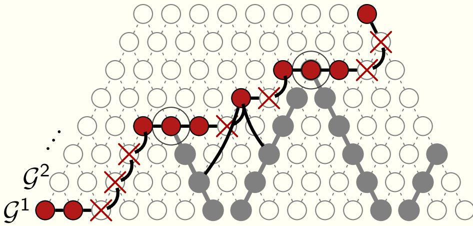

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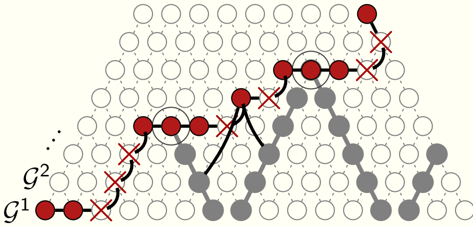

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`Temporal-Diameter` **is solvable with** $O(\delta)$ **elementary operations**

Online Algorithms

Online Algorithms

- The optimal algorithms can be adapted to an **online setting**
- The sequence of graphs $G_1, G_2, G_3, \ldots$ of $\mathcal{G}$ is processed in the order of reception
- **Amortized cost of** $O(1)$ elementary operations per graph received
- Dynamic version: consider only the recent history

Generic Framework

A Generic Framework

- Solve other problems using the same framework

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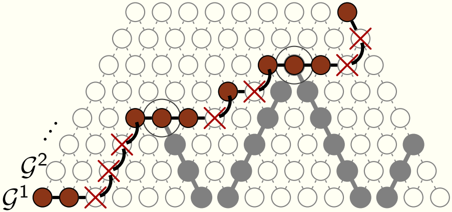

Framework generalization

- Transitive closures concatenation
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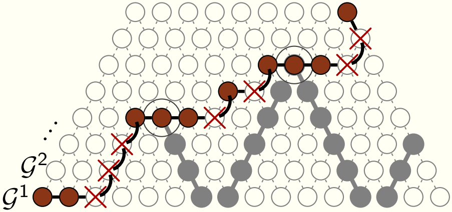

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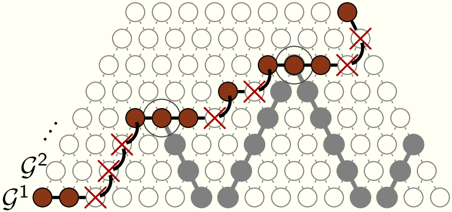

Minimization problems

Find the **smallest** value

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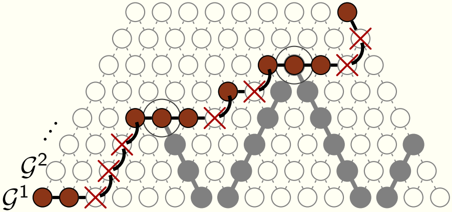

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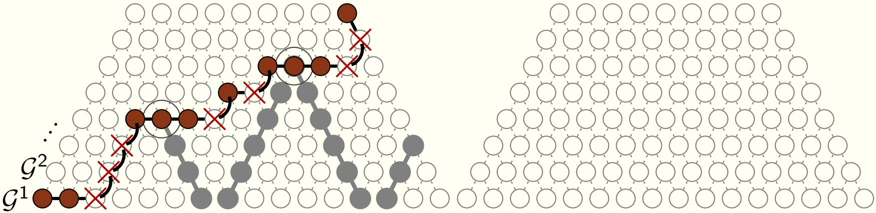

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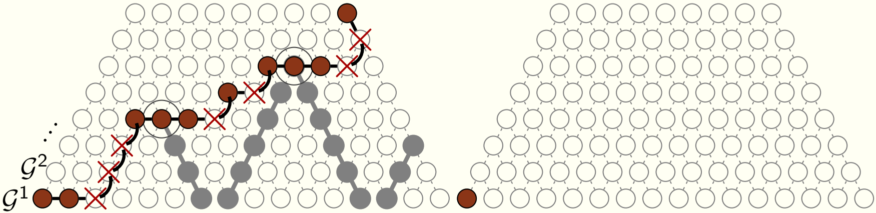

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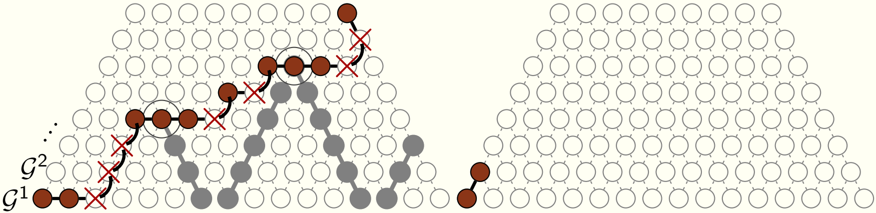

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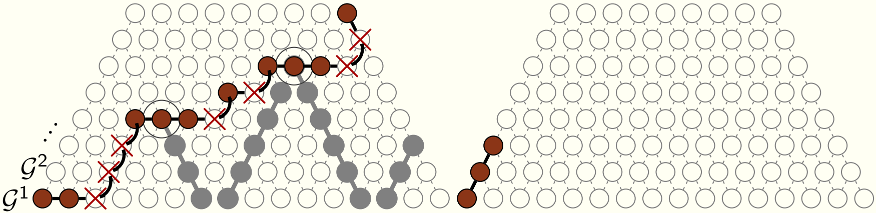

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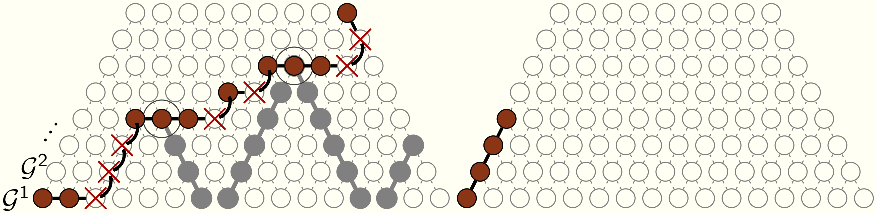

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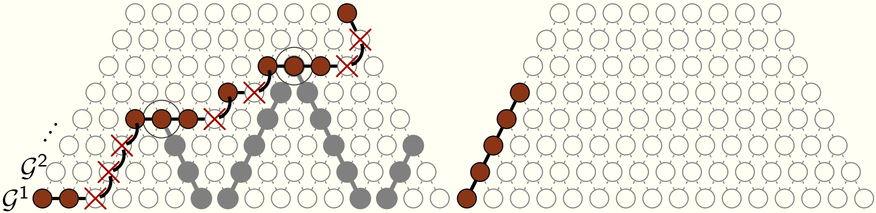

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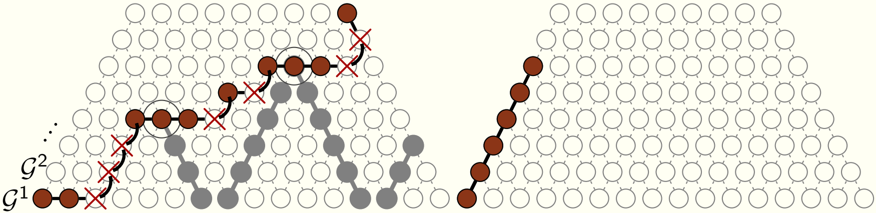

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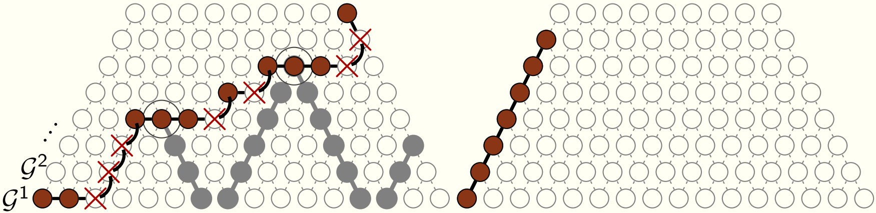

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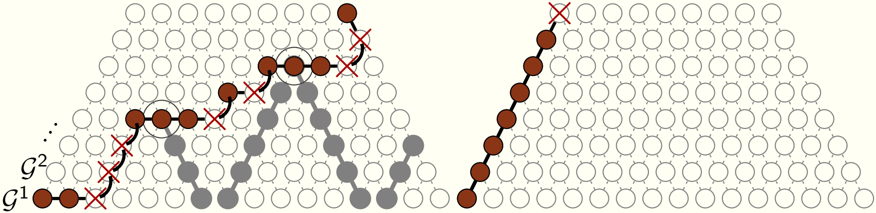

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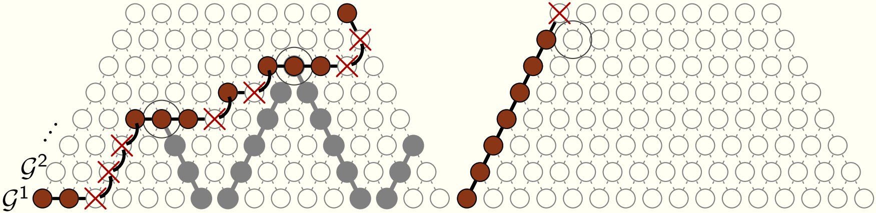

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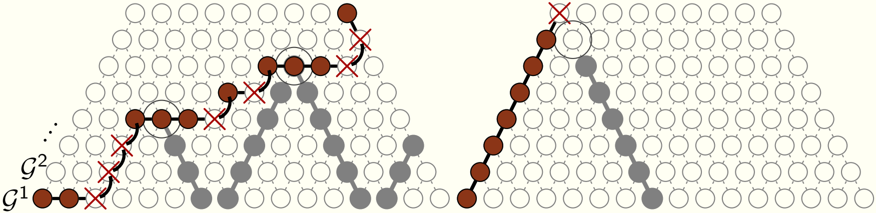

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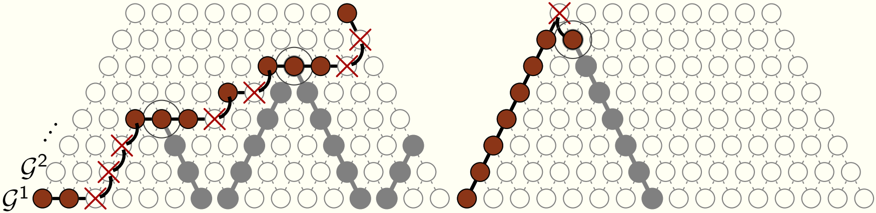

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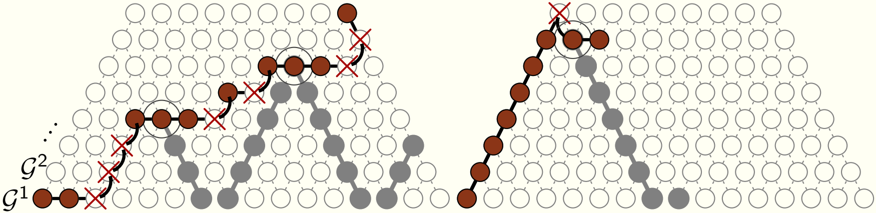

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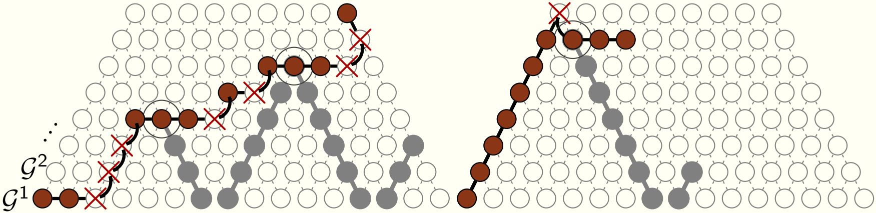

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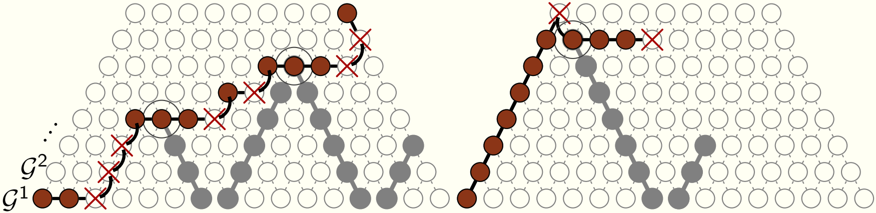

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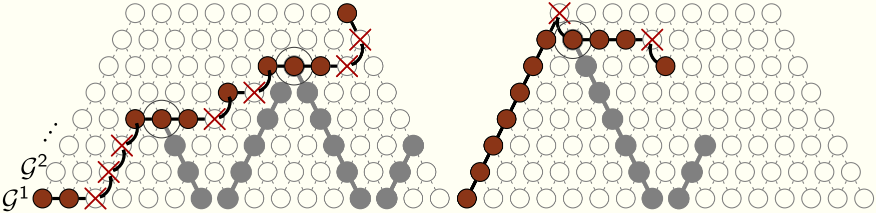

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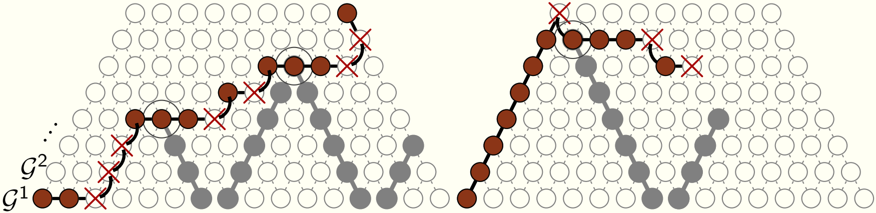

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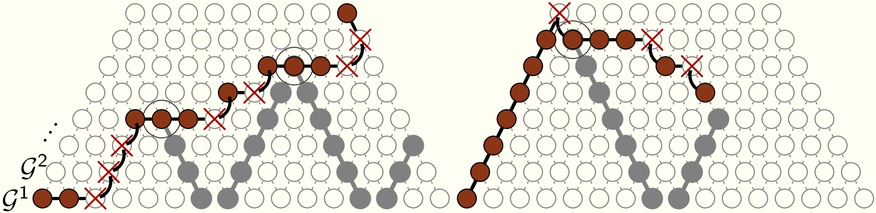

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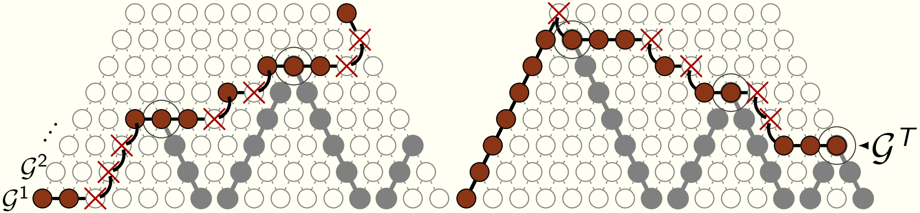

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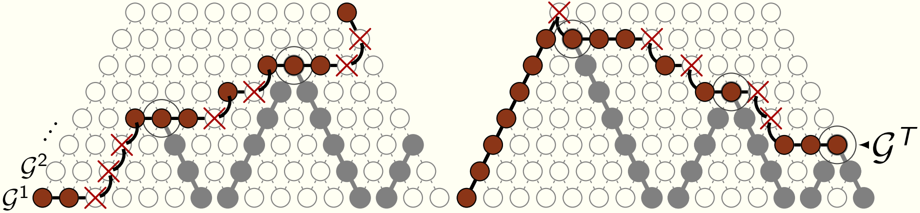

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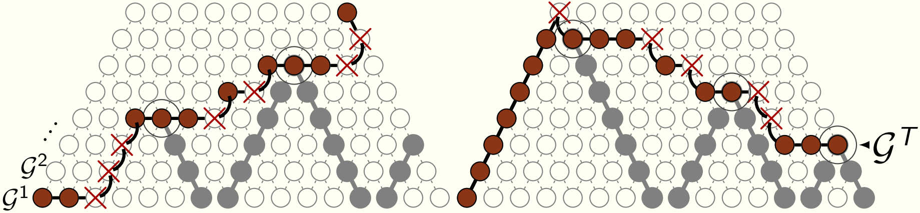

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Requirements

- $test(G_{(i,j)}) = true \Leftrightarrow \{G_i, G_{i+1}, \ldots, G_j\}$ satisfies the property P
- The composition operation is associative
- Only minimization: If $test(G_{(i,j)}) = true$ then $test(G_{(i',j')}) = true, \forall i' \leq i, j' \geq j$
- Only maximization: If $test(G_{(i,j)}) = true$ then $test(G_{(i',j')}) = true, \forall i' \geq i, j' \leq j$

Round-trip Temporal Connectivity

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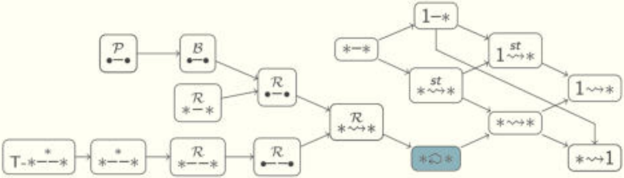
A dynamic graph $\mathcal{G}$ is round-trip temporal connected if and only if a back-and-forth journey exists from any node to all other nodes.



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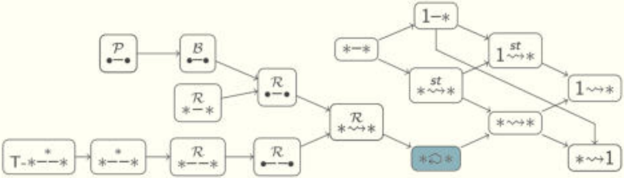
$\textsc{Round-Trip-Temporal-Diameter}$(minimization)

Finding the smallest duration in which there exists a back-and-forth journey from any node to all other nodes.



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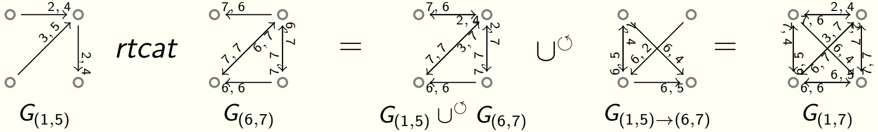
Round-Trip-Temporal-Diameter(minimization)

Finding the smallest duration in which there exists a back-and-forth journey from any node to all other nodes.

- **Super node:** Round-trip transitive closure
- **Composition operation:** Round-trip transitive closure concatenation

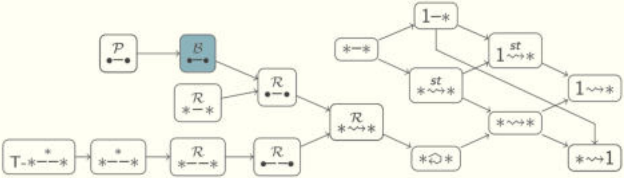


- **Test operation:** Round-trip completeness

Bounded Realization of the footprint

Time-bounded edge reappearance

A dynamic graph $\mathcal{G}$ has a time-bounded edge reappearance with a bound b if the time between two appearances of the same edge is at most b.



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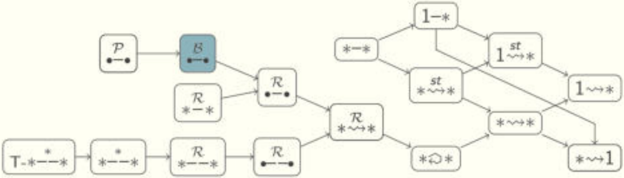
BOUNDED-REALIZATION-OF-THE-FOOTPRINT(minimization)

Finding the smallest b such that in every subsequence of length b in the sequence $\mathcal{G}$, all the edges of the footprint appear at least once.



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Bounded-Realization-of-the-Footprint(minimization)

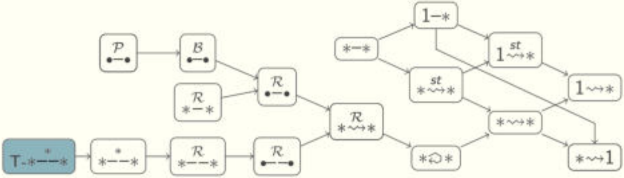
Finding the smallest b such that in every subsequence of length b in the sequence $\mathcal{G}$, all the edges of the footprint appear at least once.

- **Super node:** Union graphs
- **Composition operation:** Union
- **Test operation:** Equality to the footprint

T-interval Connectivity

Definition: T-interval connectivity

A dynamic graph $\mathcal{G}$ is T-interval connected if and only if every T length sequence of graphs has a common connected spanning sub-graph.



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Finding the largest T for which the graph is T-interval connected.

$\mathcal{G}$ 

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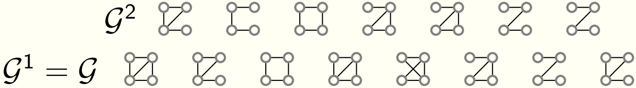

T-interval Connectivity

Definition: T-interval connectivity

A dynamic graph $\mathcal{G}$ is T-interval connected if and only if every T length sequence of graphs has a common connected spanning sub-graph.

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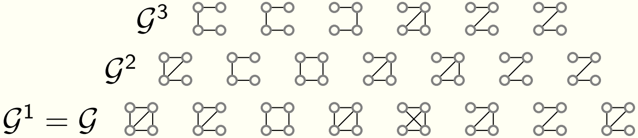

- **Super node:** Intersection graph
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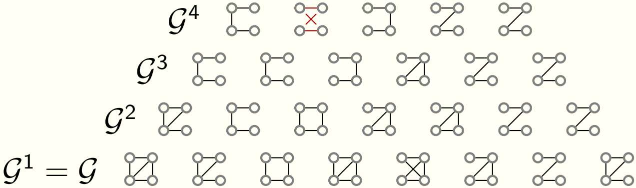

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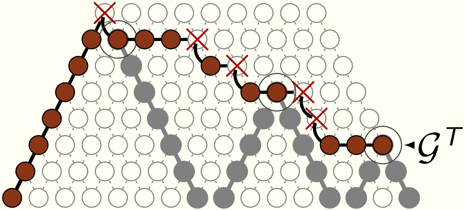

Symmetric Problems

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A minimization or maximization problem is symmetric if:
for all $i, j, i', j' \leq \delta$, $i \leq i' \leq j$, $composition(G_{(i,j)}, G_{(i',j')}) = G_{(i,j')}$.

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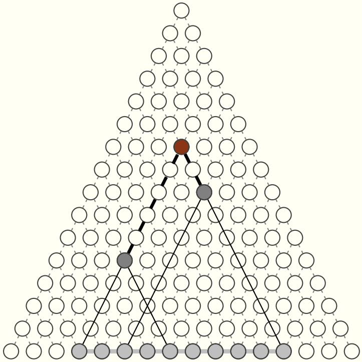

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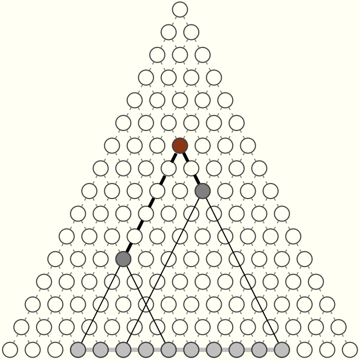

- e.g T-Interval-Connectivity and Bounded-Realization-of-the-Footprint

Row-Based Strategy

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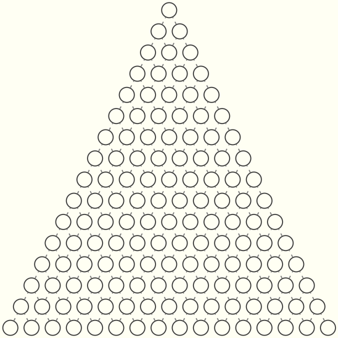
Symmetric problems (maximization)



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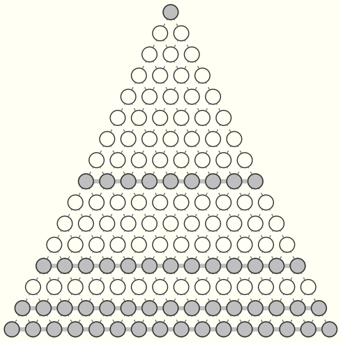

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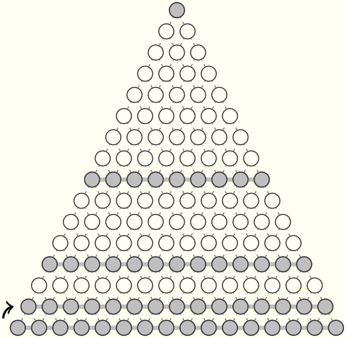

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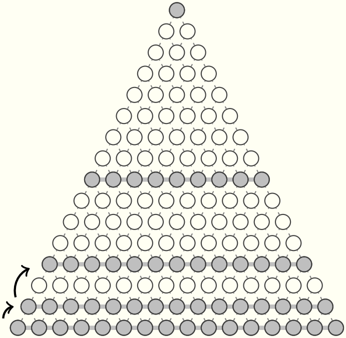
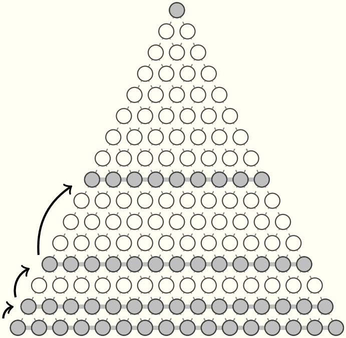


Row-Based Strategy

Symmetric problems (maximization)

- $O(\delta)$ composition per row
- $O(\delta)$ tests per row



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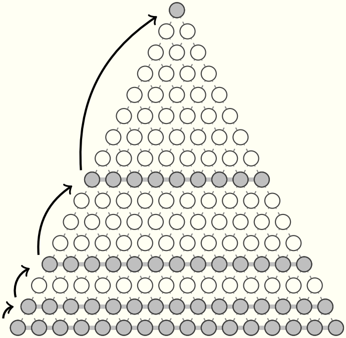

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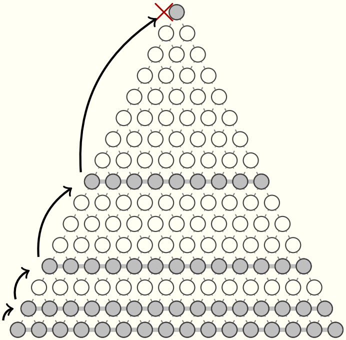

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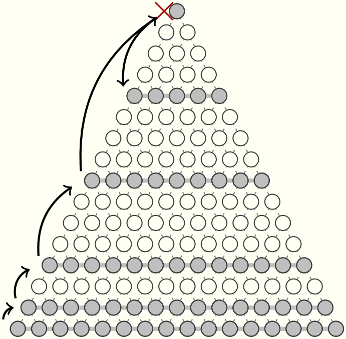

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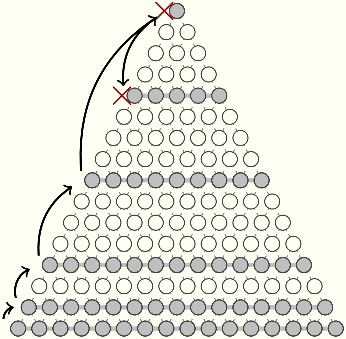

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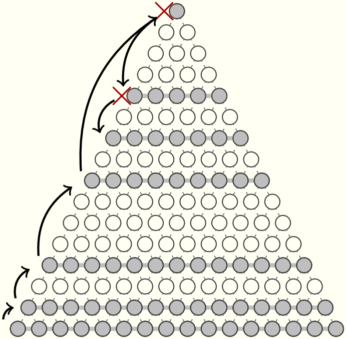

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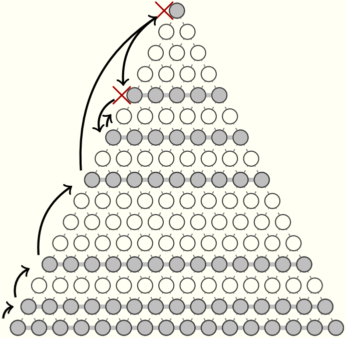

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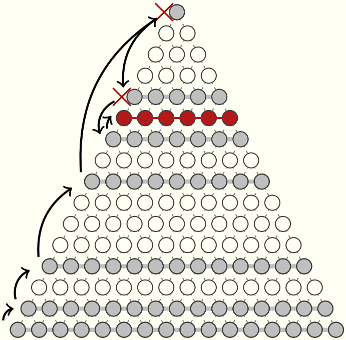

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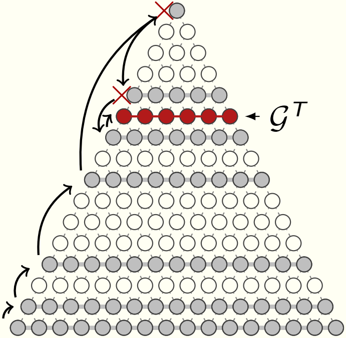
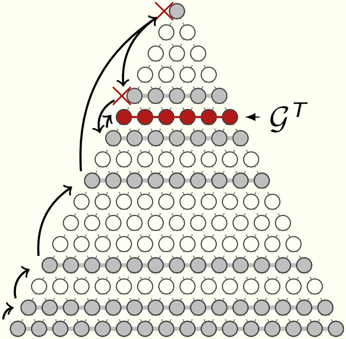


Row-Based Strategy

Symmetric problems (maximization)

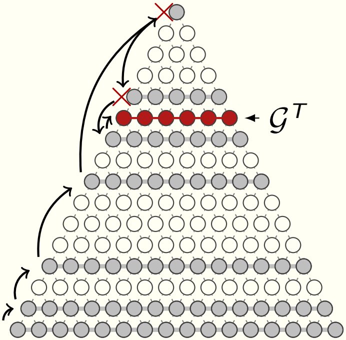
- $O(\delta)$ composition per row
- $O(\delta)$ tests per row
- $O(\log \delta)$ rows



Row-Based Strategy

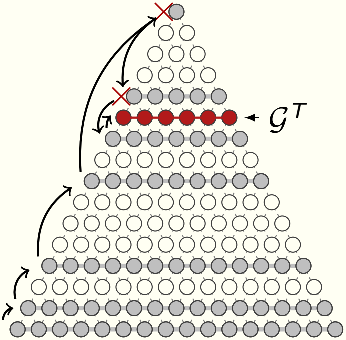
Symmetric problems (maximization)
$O(\delta \log \delta)$ **elementary operations**

- $O(\delta)$ composition per row
- $O(\delta)$ tests per row
- $O(\log \delta)$ rows



Parallel Version

- On EREW PRAM



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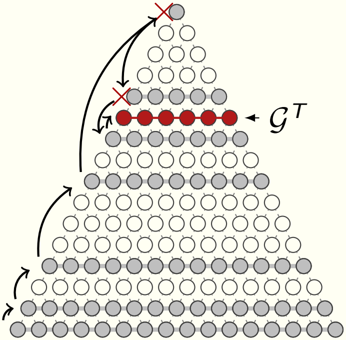

Parallel Version

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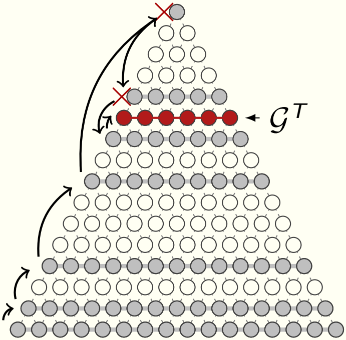
  - Symmetric problems are solvable in $O(\log^2 \delta)$ on an **EREW PRAM** with $O(\delta)$ processors



Conclusion

- Conclusion
  - High-level strategies for computing minimization and maximization parameters
  - Algorithms that use only $O(\delta)$ elementary operations
  - Parallel versions on PRAM (in Nick's class)
  - Online algorithms with amortized cost of $O(1)$ elementary operations per graph received
- Perspectives
  - How about other classes?
  - Generic Framework
    - What if the evolution of the dynamic graph is constrained?

Thank you !