On Robust Temporal Structures in Highly Dynamic Networks

Arnaud Casteigts
(LaBRI, University of Bordeaux)

J. work with Swan Dubois, Franck Petit, and John Michael Robson

https://arxiv.org/abs/1703.03190

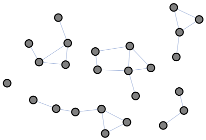
AATG@ICALP 2018

Highly dynamic networks

Ex: 

How changes are perceived?

- ~~Faults and Failures?~~
- Nature of the system. Change is normal.
- Possibly partitioned network



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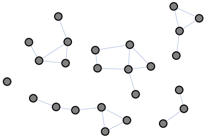
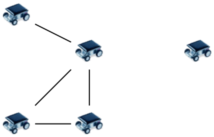


Example of scenario



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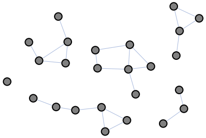

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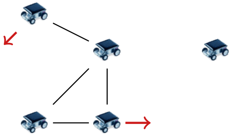

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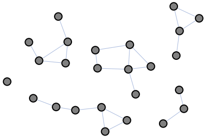

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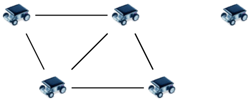

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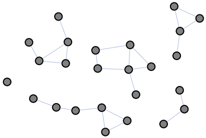

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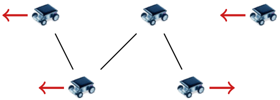

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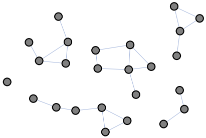

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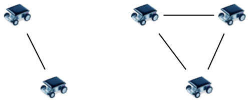

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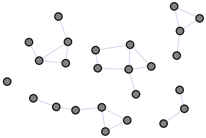

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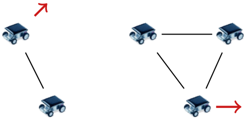

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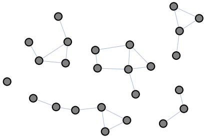

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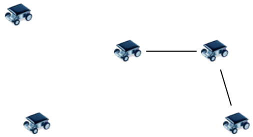

Graph representations

Time-varying graphs (TVG)

$\mathcal{G} = (V, E, \mathcal{T}, \rho, \zeta)$

- $\mathcal{T} \subseteq \mathbb{N}/\mathbb{R}$ (lifetime)
- $\rho : E \times \mathcal{T} \rightarrow \{0, 1\}$ (presence fonction)
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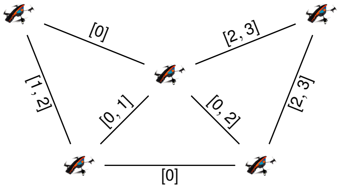
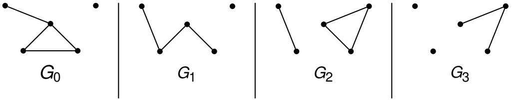


Another classical view $\mathcal{G} = G_0, G_1, \ldots$



Variety of models and terminologies:

Dynamic graphs, evolving graphs, temporal graphs, link streams, etc.

C., Flocchini, Quattrociocchi, Int. J. of Parallel, Emergent and Distributed Systems, Vol. 27, Issue 5, 2012

(among others)

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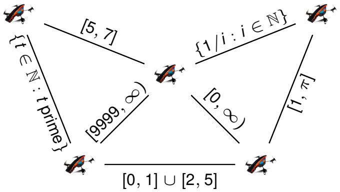

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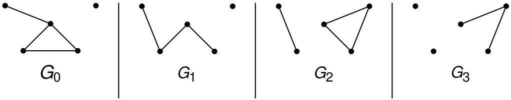

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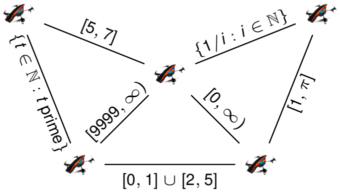

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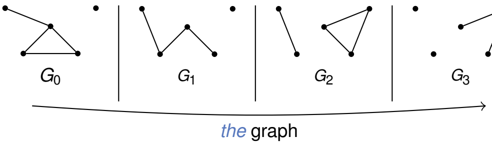

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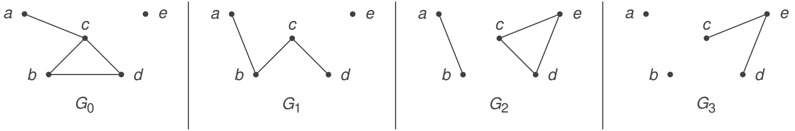
Basic concepts



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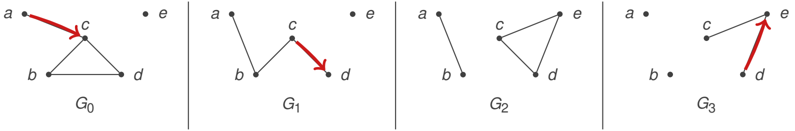

$\Longrightarrow$ *Temporal path* (*a.k.a. Journey*), *e.g.* $a \rightsquigarrow e$

Ex: $((ac, t_1), (cd, t_2), (de, t_3))$ with $t_{i+1} \geq t_i$ and $\rho(e_i, t_i) = 1$

(can be formulated with latency)

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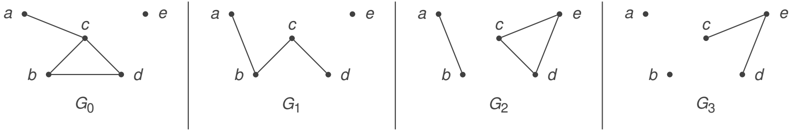

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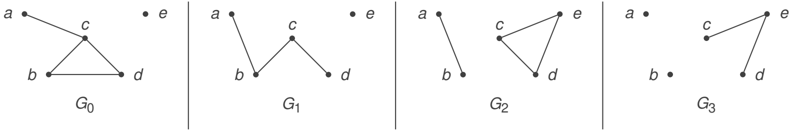

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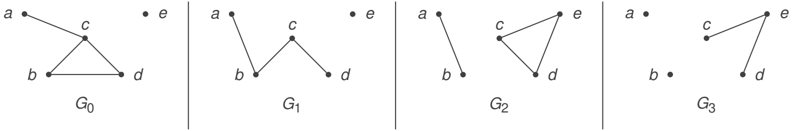

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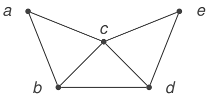
$\Longrightarrow$ *Footprint ($\neq$ underlying graph)*

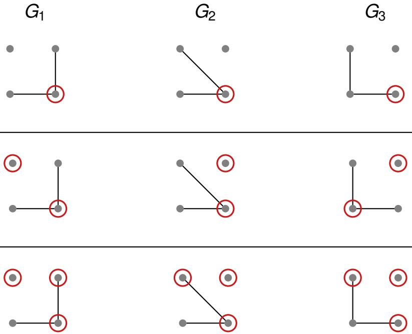


Today: Covering problems

Three ways of redefining covering problems

C., Mans, Mathieson, 2011

Ex: DOMINATINGSET



Temporal dominating set

Evolving dominating set

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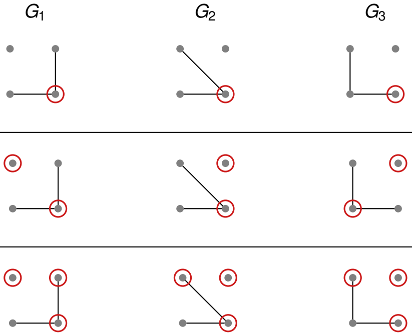
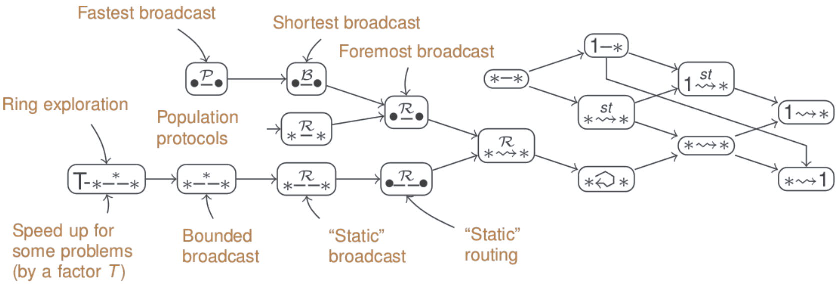


$\rightarrow$ How about infinite time? The relation must hold infinitely often!

Classes of dynamic networks (C.,Flocchini,Quattrociocchi,Santoro, 2012)

What assumption for what problem?



(based on time-varying graphs)

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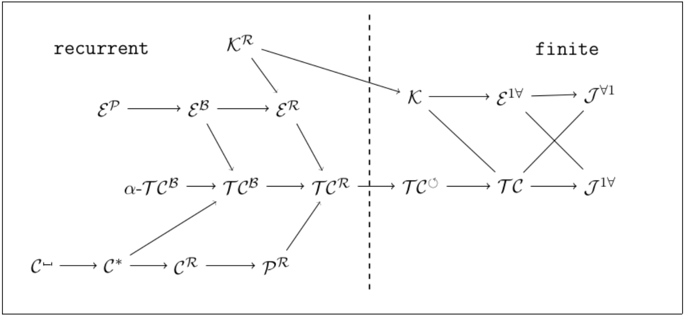

(C., 2018)

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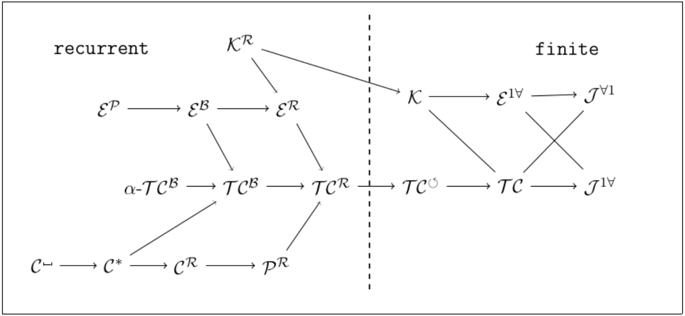

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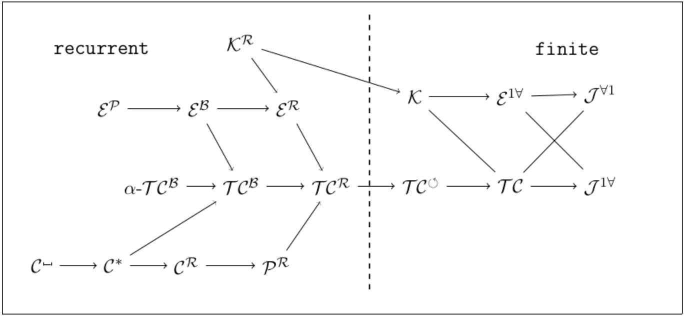

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Building temporal covering structures?

$\rightarrow$ $\mathcal{E}^{\mathcal{R}}$: “easy”
$\rightarrow$ $\mathcal{TC}^{\mathcal{R}}$: this talk

Exploiting regularities within $\mathcal{TC}^{\mathcal{R}}$

$\mathcal{TC}^{\mathcal{R}}$:= Temporal connectivity is recurrently achieved $(\mathcal{TC}^{\mathcal{R}} := \forall t, \mathcal{G}_{[t,+\infty)} \in \mathcal{TC})$

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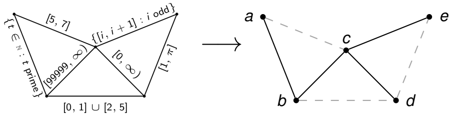
Alternative characterization: $\mathcal{TC}^{\mathcal{R}} \equiv$ Eventual footprint connected — Braud Santoni et al., 2016



$\longrightarrow$ Can be exploited in a distributed algorithm — Kaaouachi *et al.*, 2016

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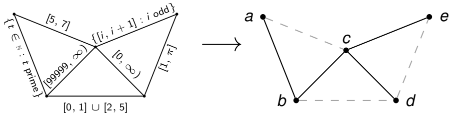


→ Can be exploited in a distributed algorithm $\quad$ Kaaouachi *et al.*, 2016

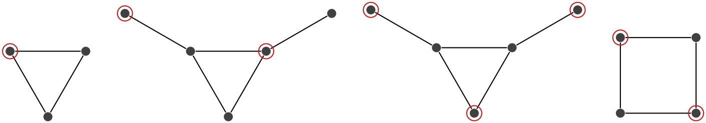
→ Robustness: New form of heredity asking that a property or solution holds in all connected spanning subgraphs

Ex: MinimalDominatingSet (MDS) and MaximalIndependentSet (MIS)

C., Dubois, Petit, Robson, 2017/18

Ex: Maximal Independent Sets

A maximal independent set (MIS) is a maximal ($\neq$ maximum) set of nodes, none of which are neighbors.



(a) (b) (c) (d)

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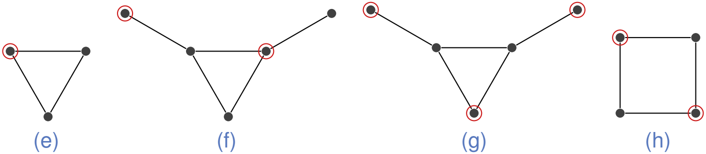

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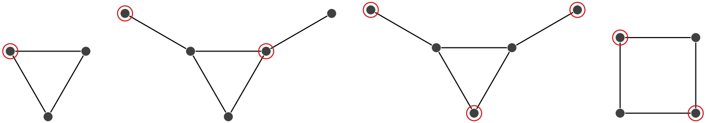


(i) (j) (k) (l)

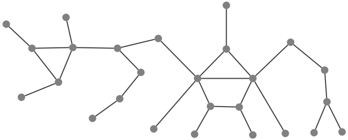
Which ones are robust?

$\rightarrow$ Question: characterizing graphs/footprints in which

1. all MISs are robust: ($\mathcal{RMIS}^{\forall}$)
2. at least one MIS is robust: ($\mathcal{RMIS}^{\exists}$)
3. all MDSs are robust: ($\mathcal{RMDS}^{\forall}$)
4. at least one MDS is robust: ($\mathcal{RMDS}^{\exists}$)

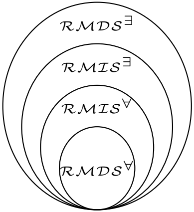
Overview of technical results

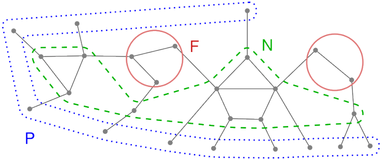
1. $\mathcal{RMDS}^\forall$ = Sputniks
2. $\mathcal{RMIS}^\forall$ = Complete bipartite $\cup$ Sputniks
3. $\mathcal{RMDS}^\exists \supsetneq$ bipartite + test algo
4. $\mathcal{RMIS}^\exists \supsetneq$ bipartite + test algo



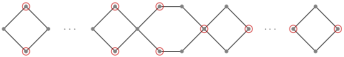
Locality:

1. $\mathcal{RMDS}^\forall$ and $\mathcal{RMIS}^\forall$
   $\rightarrow$ Robust solutions can be computed locally!
2. $\mathcal{RMIS}^\exists$
   $\rightarrow$ Robust solutions cannot be computed locally!





Local algo for robust MIS in Sputniks



Lower bound on the non-locality of robust MIS

$\mathcal{RMIS}^{\forall}$

Graphs in which *all* MISs are robust? ($\mathcal{RMIS}^{\forall}$)

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Lemma

Bipartite complete ($\mathcal{BK}$) graphs $\subseteq \mathcal{RMIS}^{\forall}$.

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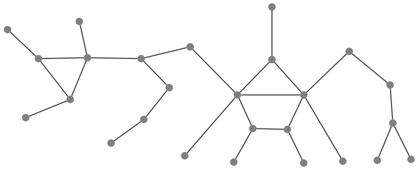

Def: A graph is a sputnik if and only if every node that belongs to a cycle also has an antenna (*i.e.* a pendant neighbor).

Lemma

Sputniks $\subseteq \mathcal{RMIS}^{\forall}$.

$\mathcal{RMIS}^\forall$

Graphs in which *all* MISs are robust? ($\mathcal{RMIS}^\forall$)

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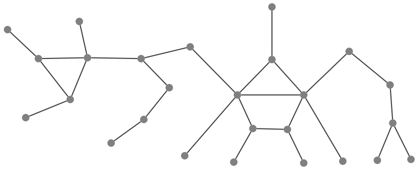

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Lemma

Sputniks $\subseteq \mathcal{RMIS}^\forall$.

Theorem

$\mathcal{RMIS}^\forall = \textit{Sputniks} \cup \mathcal{BK}$

Local algorithm to find a RMIS in $\mathcal{RMIS}^{\forall}$

State of the art (classical MIS)

- Lower bound: $\Omega(\sqrt{\log n / \log\log n})$ [KMW04]
- Best algo: $2^{O(\sqrt{\log n})}$ [PS96] (between $\log n$ and n)
- Best algo in trees: $O(\log n / \log\log n)$ [BE10]

Can we solve the problem locally in $\mathcal{RMIS}^{\forall}$?

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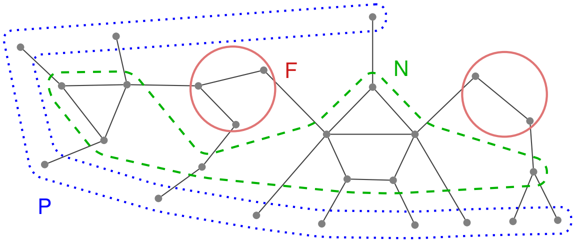

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Local algorithm to find a RMIS in $\mathcal{RMIS}^{\forall}$

State of the art (classical MIS)

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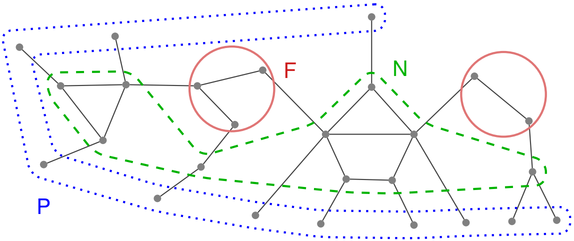

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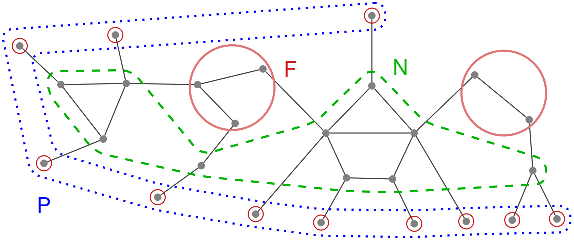

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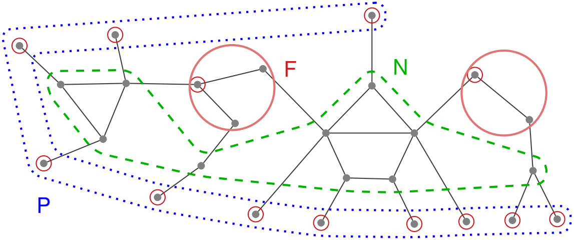

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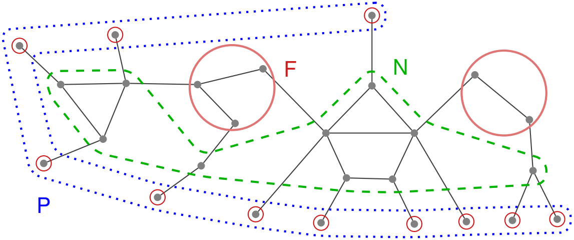

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$\Longrightarrow$ $\boxed{o(\log n)}$

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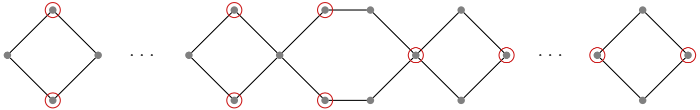

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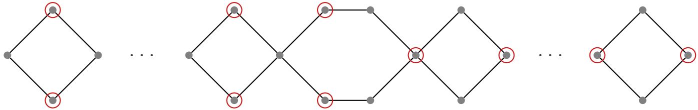

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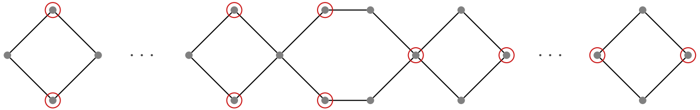

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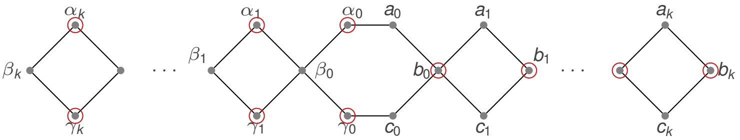

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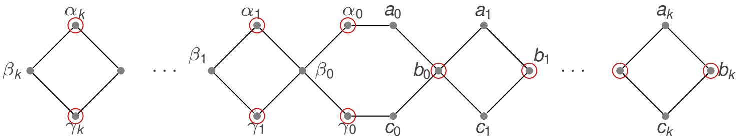

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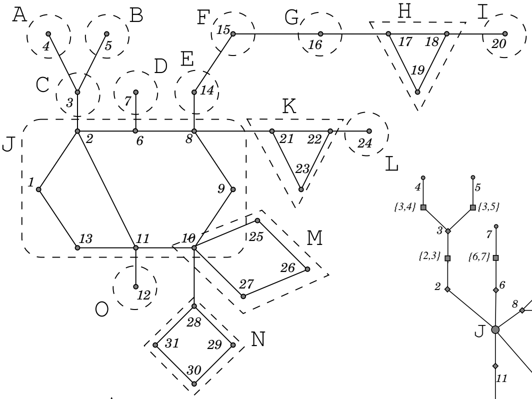
$\rightarrow$ Essentially as bad as collecting all information at one node and use offline algo.

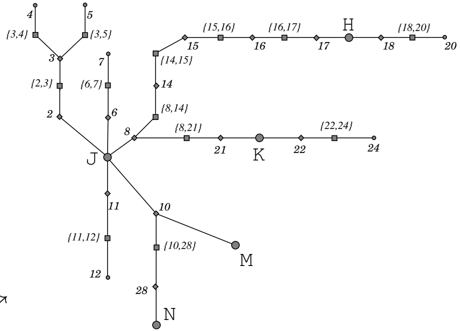
Centralized algorithm to find RMISs in general (in **P**)

Objective: Finds a RMIS if one exists, rejects otherwise.

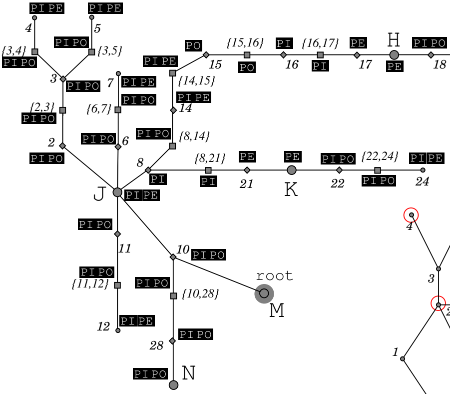


↑
Decomposition into biconnected components

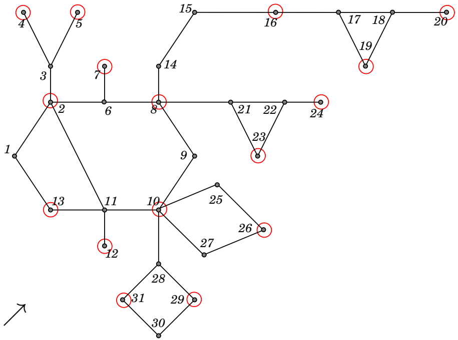
$\mathcal{ABC}$-tree ↗



Polynomial-time algorithm to find RMISs (2)



↑
Tagging



Resulting RMIS ↗

Děkuji !