Decentralized Computations by Mobile Agents in Time-Varying Graphs

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Decentralized Computations by Mobile Agents in Time-Varying Graphs

joint work with

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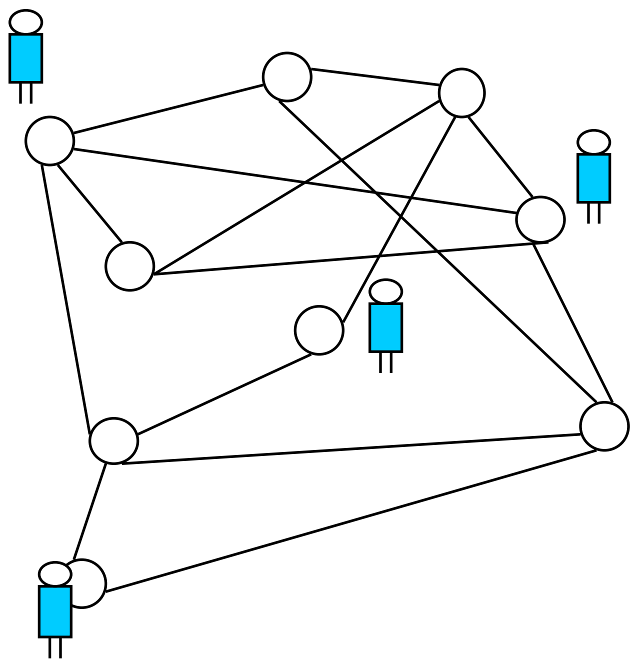
G. Prencipe, N. Santoro, G. Viglietta

Paola Flocchini - Prague 2018

DISTRIBUTED COMPUTING by COMPUTATIONAL ENTITIES

OPERATE AND **MOVE** IN A **DISCRETE** SPACE



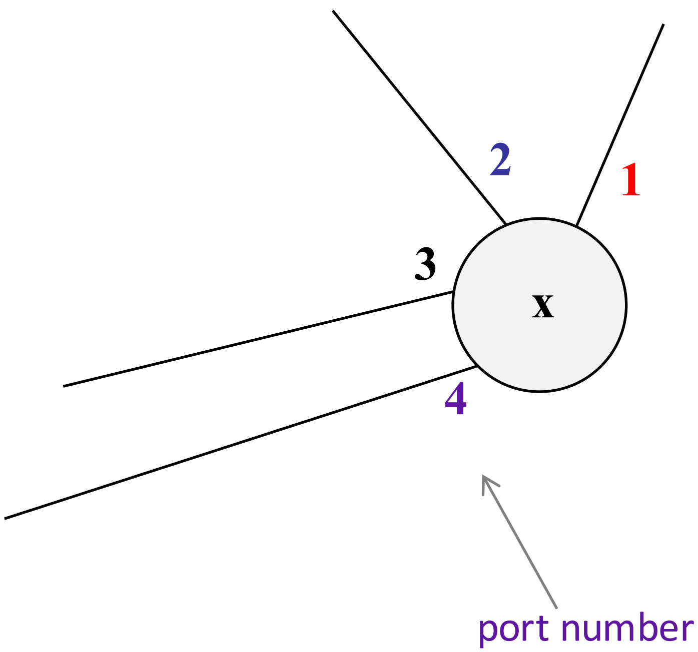
Graph G = (V , E)

V **nodes (sites , hosts)**

E **edges (links , channels)**

called agents or **robots**

Discrete Space

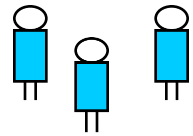


(G , λ)

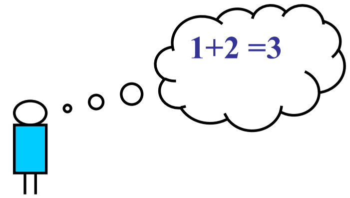
edge-labelled

Each node has a distinct **label** for its links

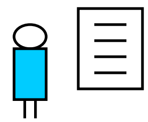
Each Agent



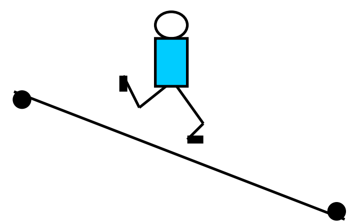
Has computing capabilities



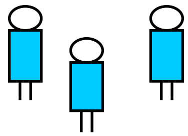
Has limited storage



Can move from node to neighboring node



The Agents



Have the same behavior **(execute the same protocol)**

Collectively

they perform some task **(solve a problem)**

Tasks / Problems

RendezVous/ Gathering

Exploration/ Map Construction

Black Hole Search

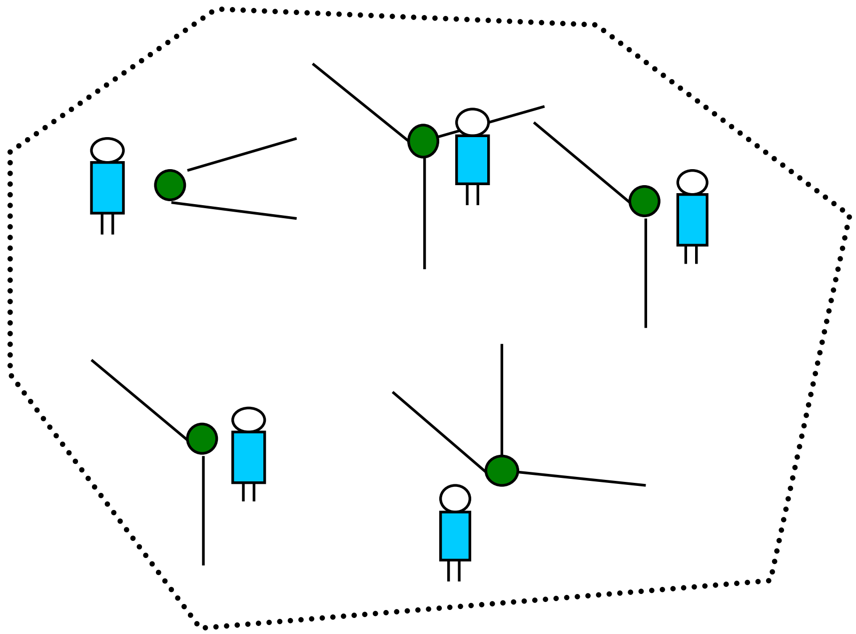
Decontamination

...

Tasks / Problems

RendezVous

Gathering

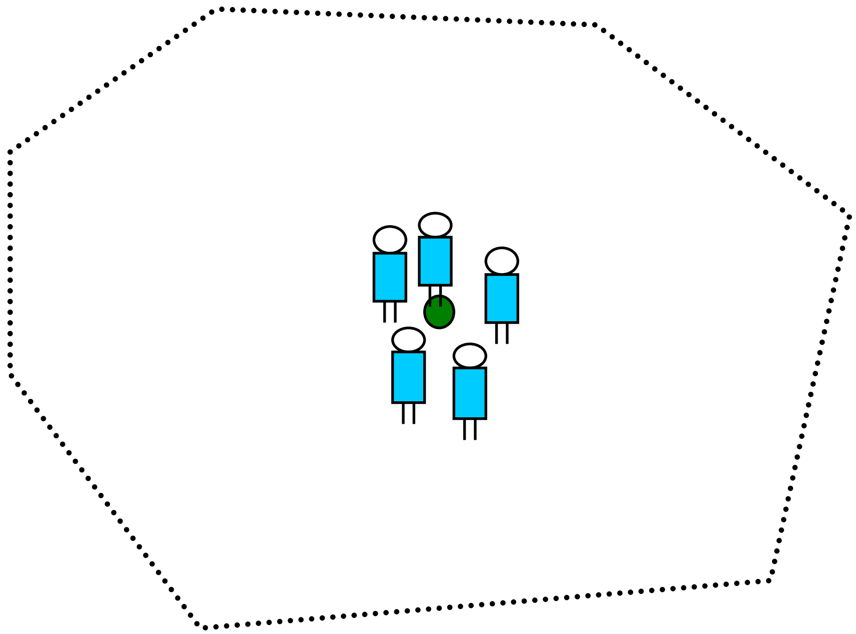


Tasks / Problems

RendezVous

Gathering

- **strict**



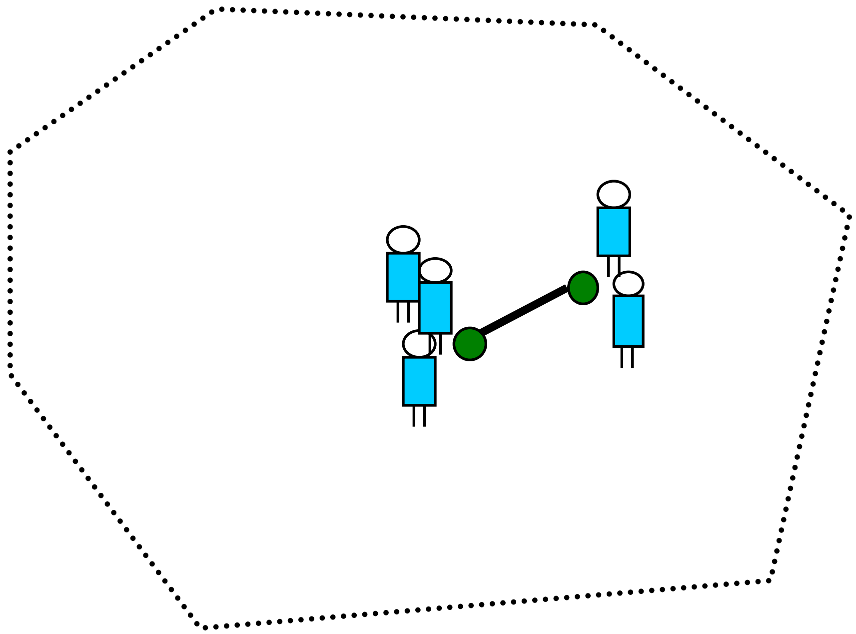
Tasks / Problems

RendezVous

Gathering

- strict

- near



Gathering in Discrete Space

Baston, Gal [*Naval Log. Res.* 91]
Alpern [*SIAM J Cont. Optimization 95*]
Anderson, Weber [*J Applied Probability 99*]
Yu, Yung [ICALP 96]
Alpern, Boston, Essegarer [*J Appl. Probability 99*]
Howard et al [*Operation research 99*]
Barrière, Flocchini, Fraigniaud, Santoro [SPAA 03]
Dessmark, Fraigniaud, Pelc [ESA 03]
Dobrev, Flocchini, Prencipe, Santoro [OPODIS 03]
Kranakis, Krizac, Santoro, Sawchuk [ICDCS 03]
Kowalski, Pelc [ISAAC 04]
Flocchini, Kranakis, Krizac, Santoro, Sawchuk [LATIN 04]
Dessmark, Fraigniaud, Kowalski, Pelc [*Networks '06*]
Kranakis, Krizank, Marcou [LATIN 06]

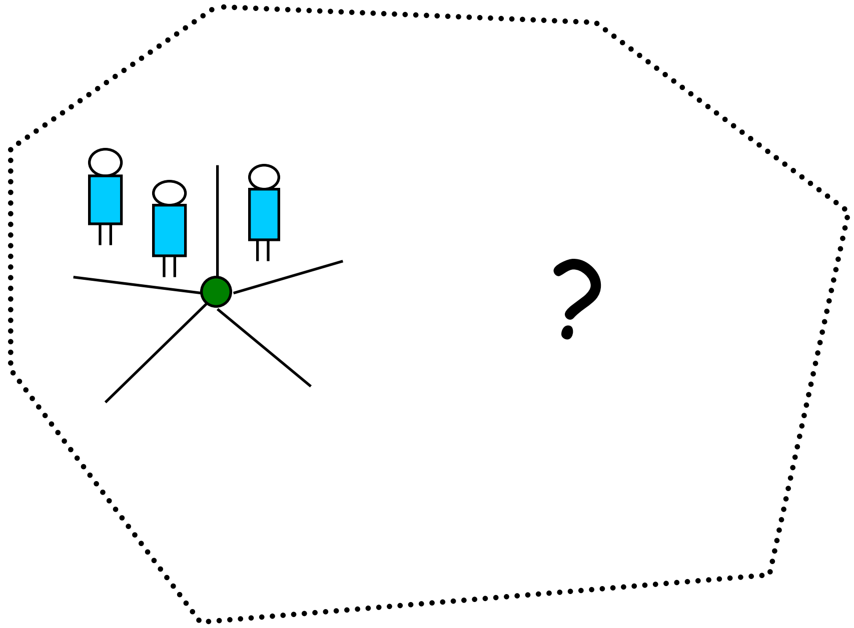
Gathering in Discrete Space

Barrière,Flocchini,Fraigniaud,Santoro [*Theo.Comp.Sys. '07*]
Chalopin [*Theo.Comp.Sci. '08*]
Czyzowicz, Dobrev, Kranakis, Krizanc [SOFSEM 08]
Klasing, Markou, Pelc [*Theo.Comp.Sci. '08*]
Kowalski, Mailnowski [*Theo.Comp.Sci. '08*]
Czyzowicz, Pelc, Labourel [ACM *Trans. Alg. '13*]
D'Angelo, Di Stefano, Klasing, Navarra [*Theo.Comp.Sci. '14*]
Das, Luccio, Markou [ALGOSENSORS 15]
Dieudonne, Pelc, Villain [*SIAM J. Comp. '15*]
Das, Luccio, Focardi, Markou, Moro, Squarcina [ICTCS 16]
Miller, Pelc [*Dist. Comput. '16*]
Bouchard, Dieudonne, Ducourthial [*Dist. Comput. '16*]
Dieudonne, Pelc [*Algorithmica '16*]
De Marco,Gargano,Kranakis,Krizank,Pelc,Vaccaro [*Theo. Comp. Sci. '16*]
E. Kranakis, D. Krizanc, E. Marcou *The Mobile Agent Rendezvous Problem in the Ring* Morgan & Claypool, 2010

AND MANY MORE …

Tasks / Problems

Exploration



Exploration/Map Contruction

Shannon [JMF 51]
Blum, Kozen [FOCS 78]
Dudek, Jenkin, Milios, Wilkes [Robotics and Automation 91]
Bender, Slonim [FOCS 94]
Betke, Rivest, Singh [Machine Learning 95]
Bender, Fernandez, Ron, Sahai, Vadhan [STOC 98]
Deng, Papadimitriou [J. Graph Theory 99]
Panaite, Pelc [J. Algorithms 99]
Awerbuch, Betke, Rivest, Singh [Information and Comp. 99]
Panaite, Pelc [Networks 00]
Albers, Henzinger [SIAMJC 00]
Duncan, Kobourov, Kumar [SODA 01]

Exploration/Map Contruction

Diks, Fraigniaud,Kranakis, Pelc [J Algorithms 02]
Fraigniaud, Ilcinkas [STACS 04]
Fraigniaud, Ilcinkas, Peer, Pelc, Peleg [MFC S04]
Das, Flocchini, Nayak, Santoro [ISAAC 06]
Gasienic,Klansing,Martin,Navarra,Zhang [SIROCCO 07]
Das, Flocchini, Kutten, Nayak, Santoro [TCS 07]

AND MANY MORE …

Gathering and Exploration in Discrete Space

Variety of assumptions and conditions

- Agents with/without ids
- Nodes with/without ids

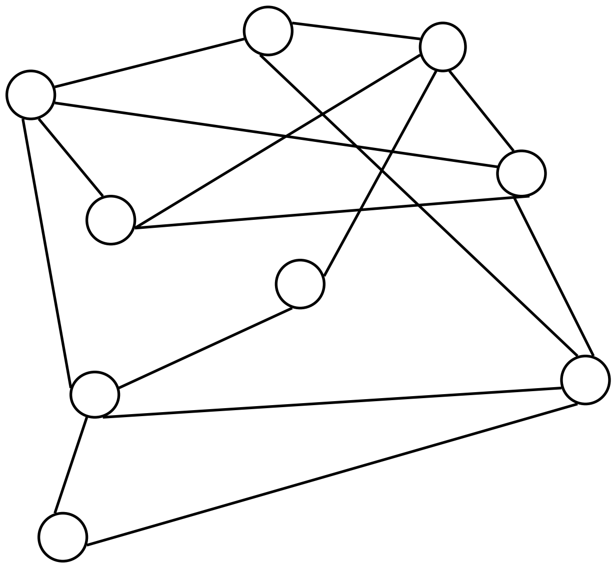
- With/without orientation

- With/without tokens

- With/without faults

- A-priori knowledge of number of agents k
- A-priori knowledge of number of nodes n
- A-priori knowledge of network topology
- …

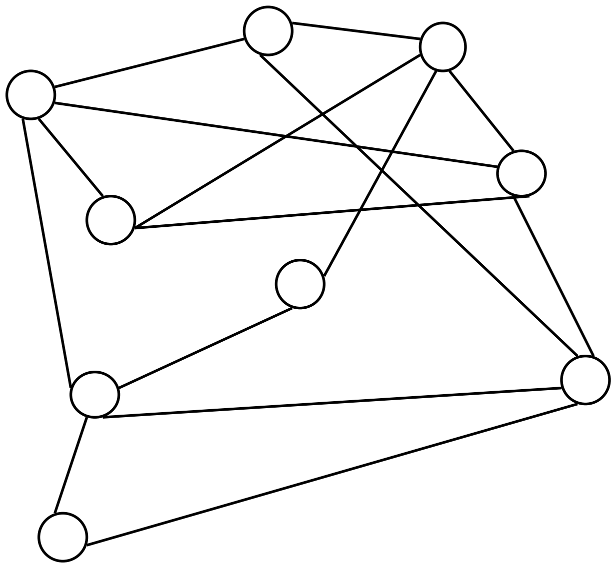
Gathering and Exploration in Discrete Space



SHARED ASSUMPTION:

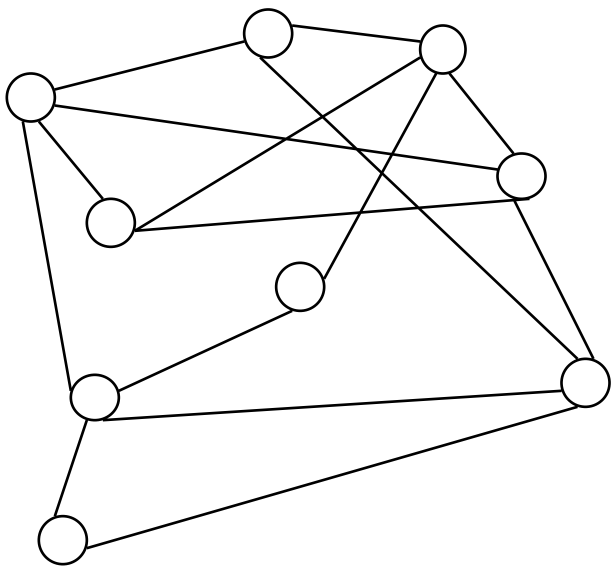
network is static

Gathering and Exploration in Discrete Space



network is dynamic

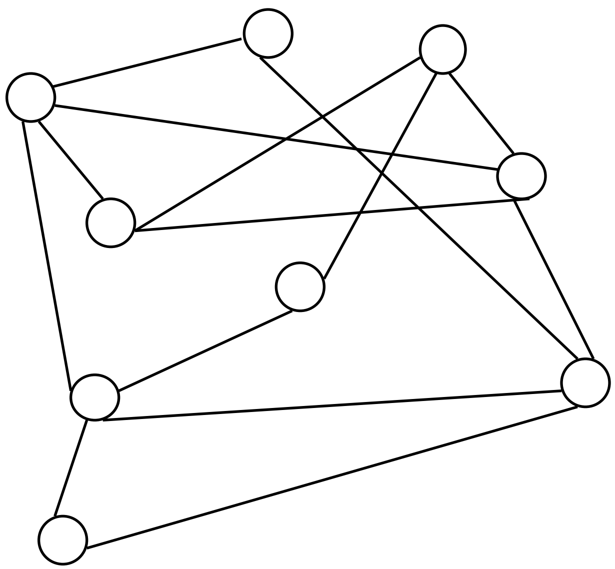
Dynamic Networks



network is dynamic

topology changes
continuously & unpredictably

Dynamic Networks



network is dynamic

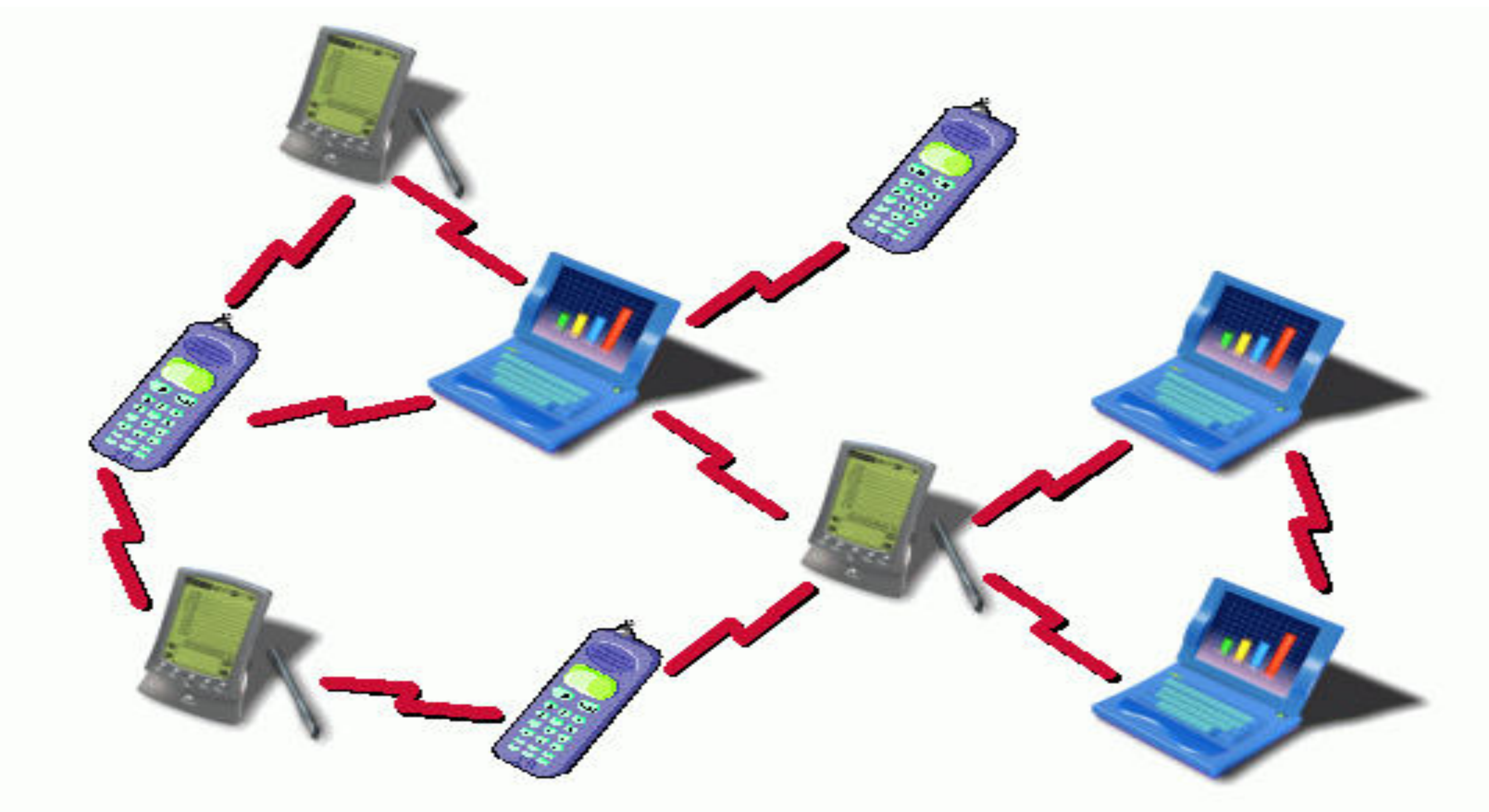
topology changes
continuously & unpredictably

(under the control of an adversary)

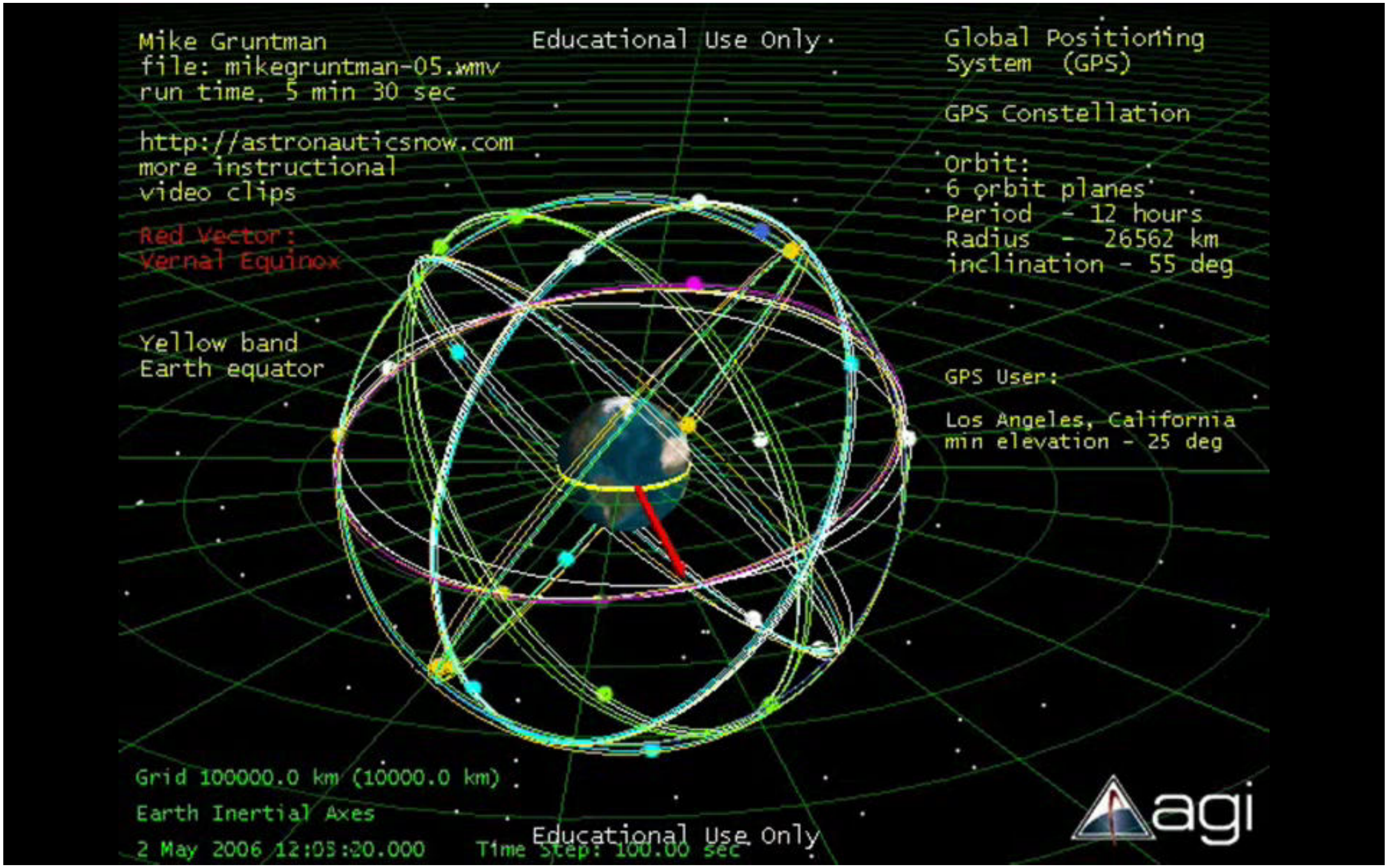
possibly disconnected

Dynamic Networks : WIRELESS MOBILE

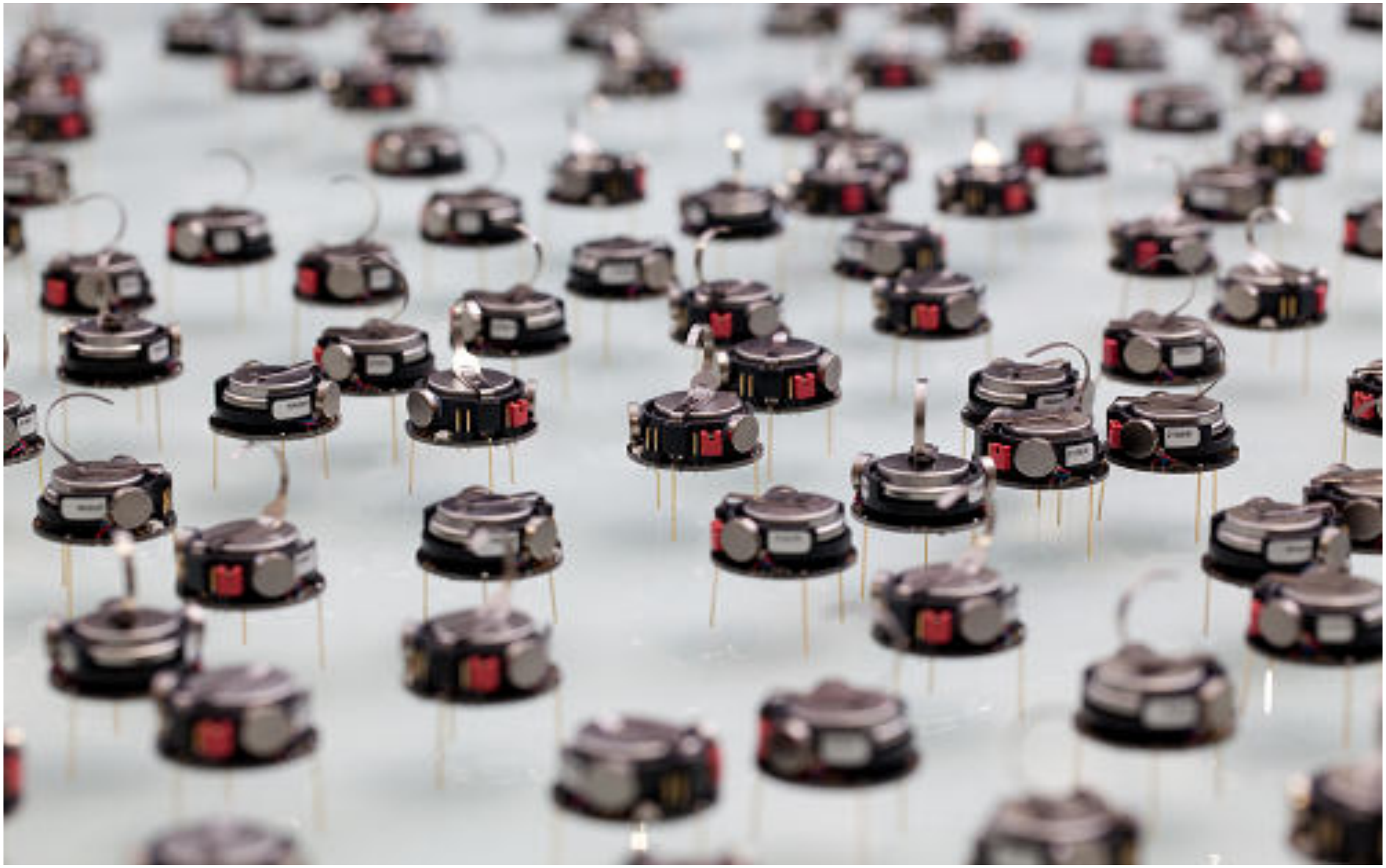
mobile ad hoc networks (MANETS)



Dynamic Networks : LEO SATELLITE NETWORK

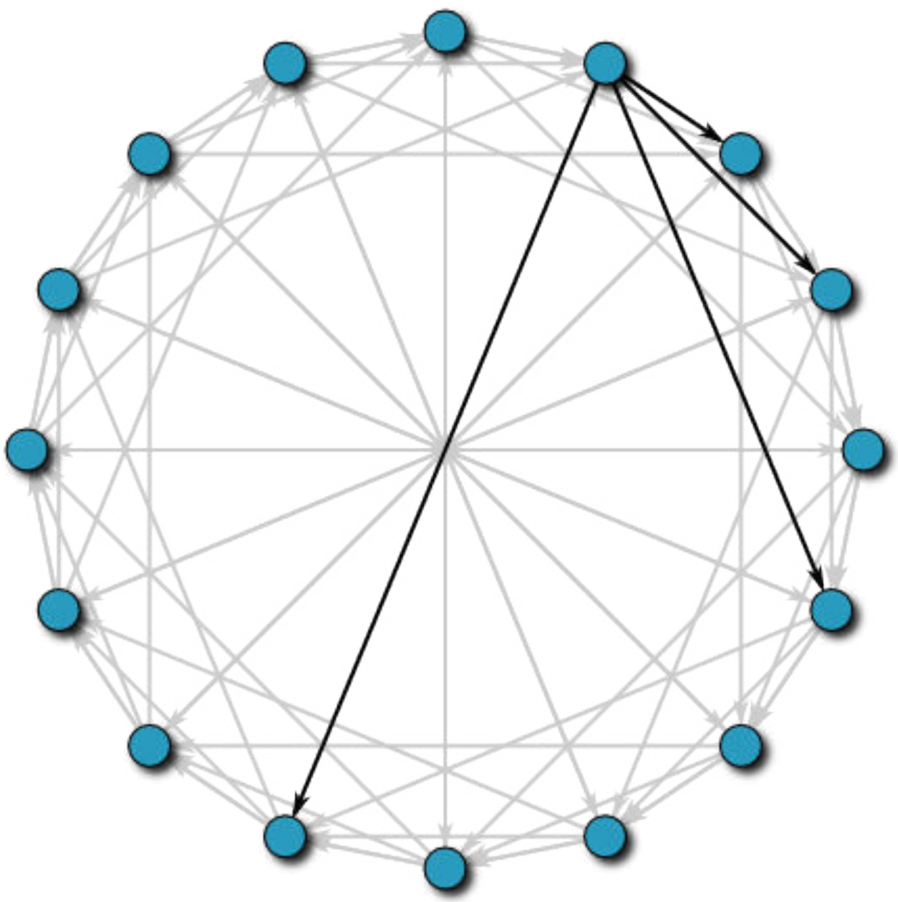


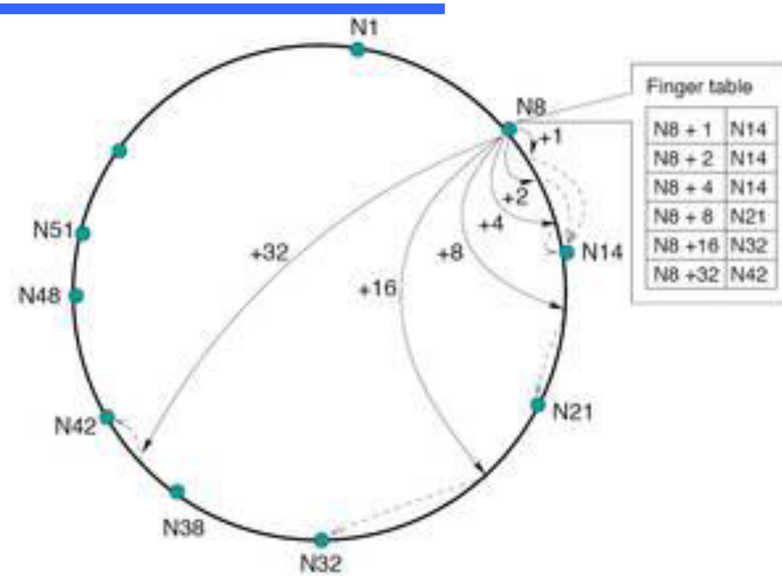
Dynamic Networks : ROBOTIC SWARMS

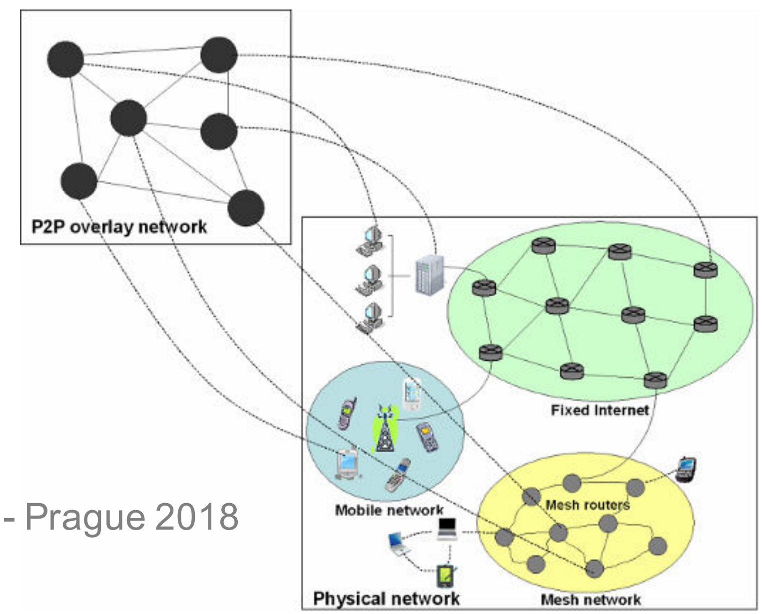


Dynamic Networks : PEER-TO-PEER

OVERLAY networks

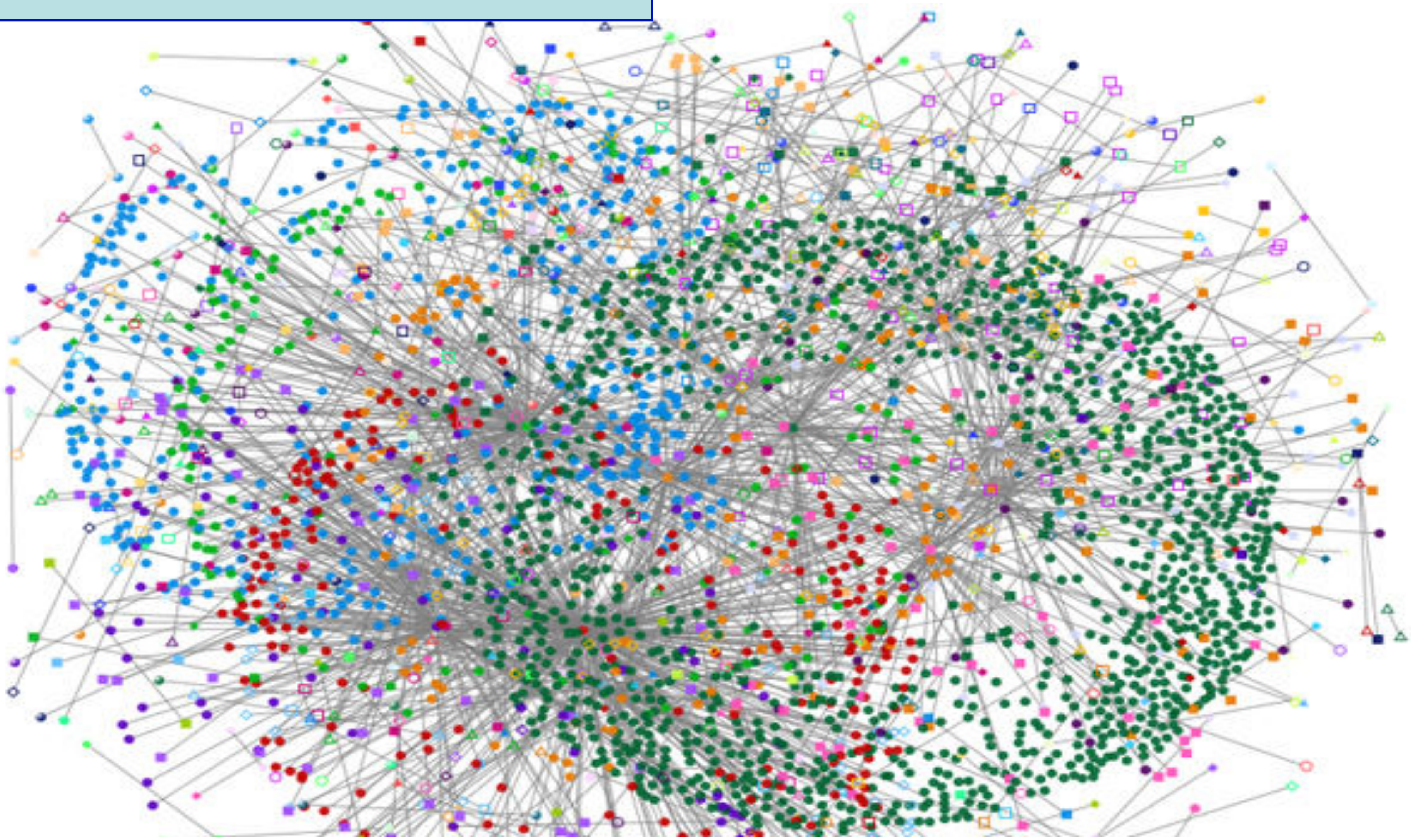






Dynamic Networks : SOCIAL NETWORKS/WEB GRAPHS

HOTMAIL



Dynamic Network

Modeled as TIME-VARYING GRAPH

A. Casteigts, P. Flocchini, W. Quattrociocchi, N.Santoro.
"Time-varying graphs and dynamic networks". *IJPEDS*, 2012

A general mathematical formalism that describes many different types of dynamic networks

A model that includes most existing models as special cases

Time-Varying Graph

$\mathcal{G} = (N, E, T, \psi, \rho, \zeta)$

Time-Varying Graph

$\mathcal{G} = (\mathbf{N}, E, T, \psi, \rho, \zeta)$

↑

nodes

Time-Varying Graph

$\mathcal{G} = (N, \mathbf{E}, T, \psi, \rho, \zeta)$

↑

edges

$E \subseteq N \times N$

Time-Varying Graph

$\mathcal{G}$ = (N, E, **T**, ψ, ρ, ζ)

↑

lifetime of system (contiguous time span)

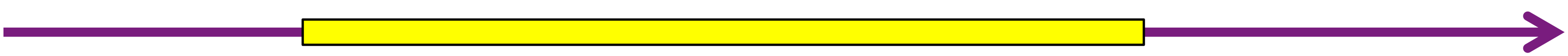
$\mathbf{T} \subseteq \mathcal{R}$

Time-Varying Graph

$\mathcal{G} = (N, E, \mathbf{T}, \psi, \rho, \zeta)$

lifetime of system (contiguous time span)

$\mathbf{T} \subseteq \mathcal{R}$



Limited (finite)

Time-Varying Graph

$\mathcal{G} = (N, E, \mathbf{T}, \psi, \rho, \zeta)$

↑

lifetime of system (contiguous time span)

$\mathbf{T} \subseteq \mathcal{R}$

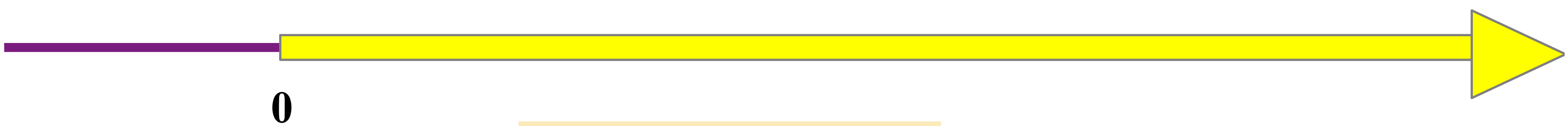
Unlimited (infinite)

Time-Varying Graph

$\mathcal{G} = (N, E, \mathbf{T}, \psi, \rho, \zeta)$

lifetime of system (contiguous time span)

$\mathbf{T} \subseteq \mathcal{R}$



Unlimited (infinite)

beginning of time line

Time-Varying Graph

$\mathcal{G} = (N, E, T, \boldsymbol{\psi}, \boldsymbol{\rho}, \zeta)$

node presence function

$\psi : N \times T \rightarrow \{0, 1\}$

$\boldsymbol{\psi}(\mathbf{x},\mathbf{t})=1$ iff
x is in present at time **t**

edge presence function

$\rho : E \times T \rightarrow \{0, 1\}$

$\boldsymbol{\rho}(\mathbf{e},\mathbf{t})=1$ iff
e is present at time **t**

Time-Varying Graph

$\mathcal{G} = (N, E, T, \psi, \rho, \boldsymbol{\zeta})$

latency (duration) function $\zeta : E \times T \rightarrow \mathbb{T} \cup \{\perp\}$

$\zeta((x,y), t) = d$

message from x to y, sent at time t, will arrive at time t+d

$\zeta((x,y), t) = \perp$

message from x to y, if sent at time t, will not arrive

Time-Varying Graph: Snapshot & Footprint

$\mathcal{G} = (N, E, T, \psi, \rho, \zeta)$

$G(t) = (N(t), E(t))$ **SNAPSHOT** at time $t \in T$

$N(t) = \{ x \in N : \psi(x, t)=1 \}$

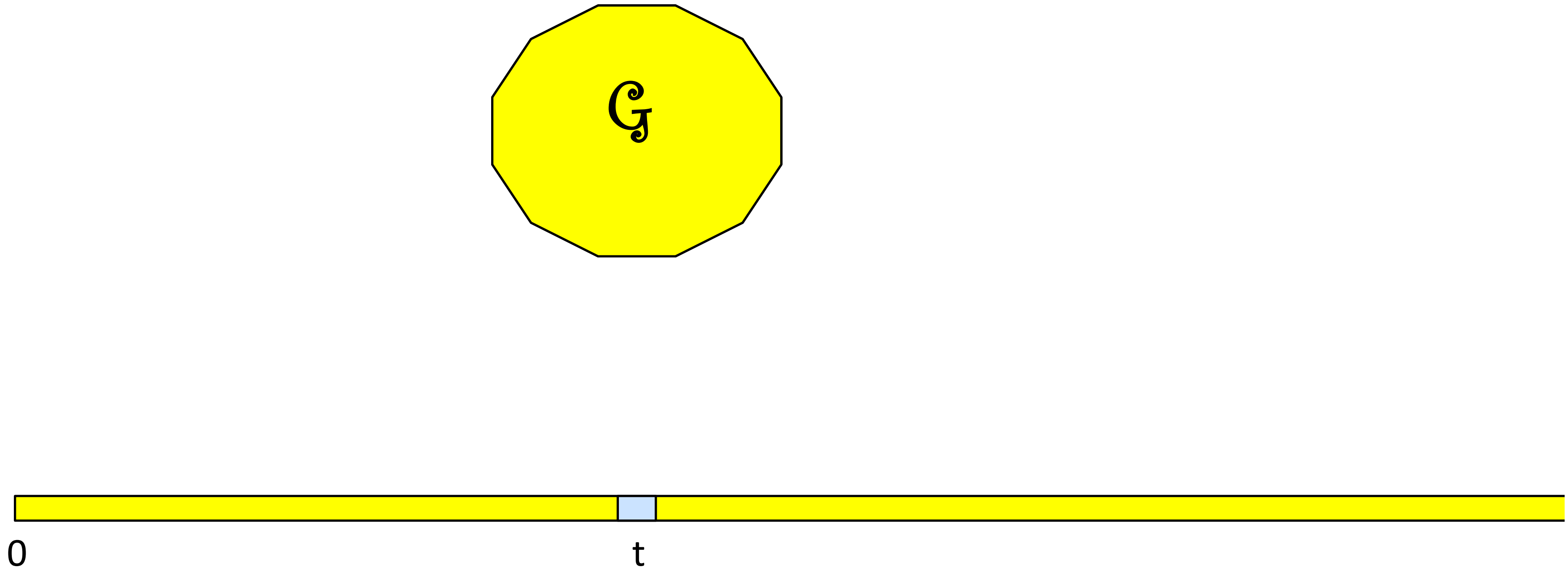
$E(t) = \{ e \in E : \rho(e, t)=1 \}$

$G = (N, E)$ **FOOTPRINT** (underlying graph)

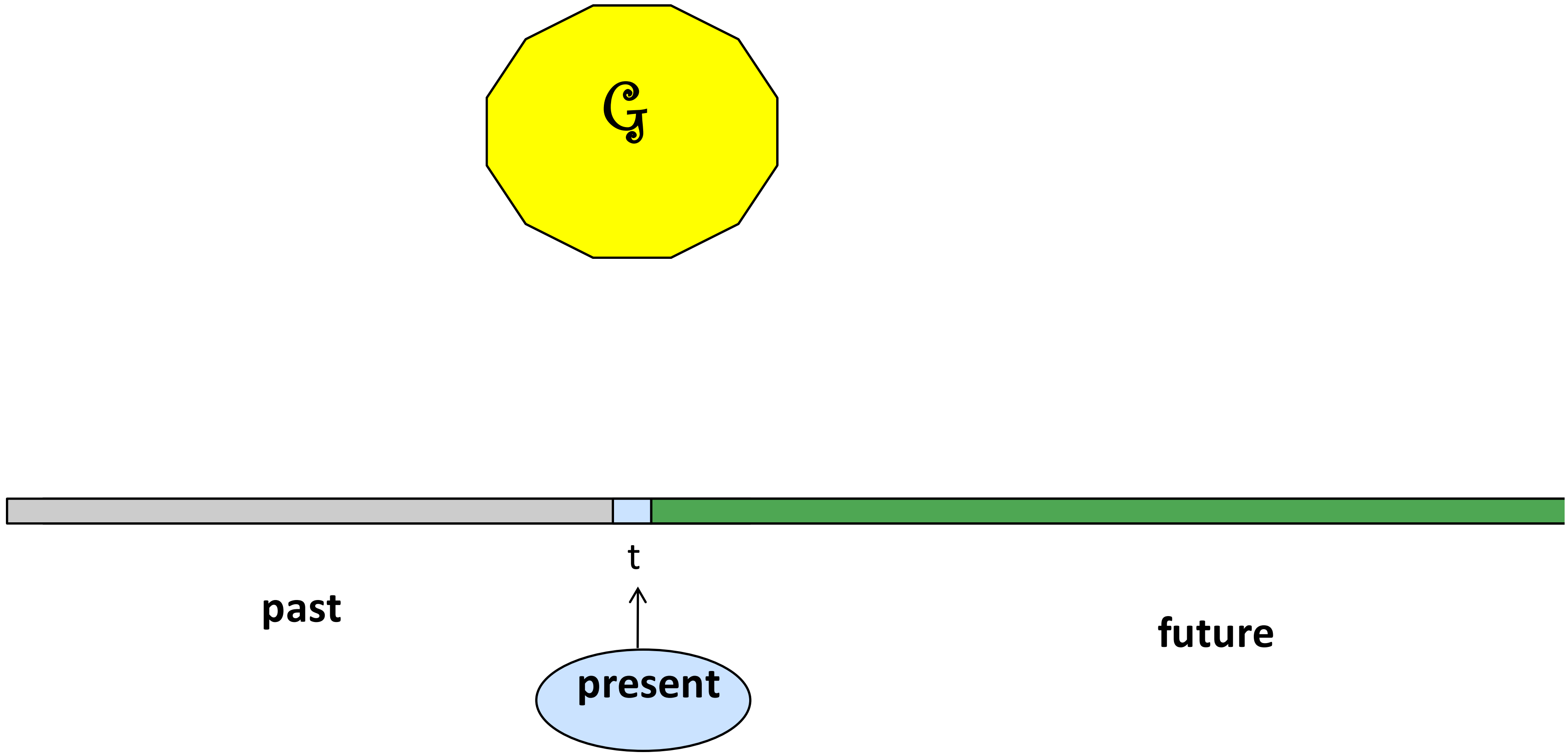
a-temporal

aggregate

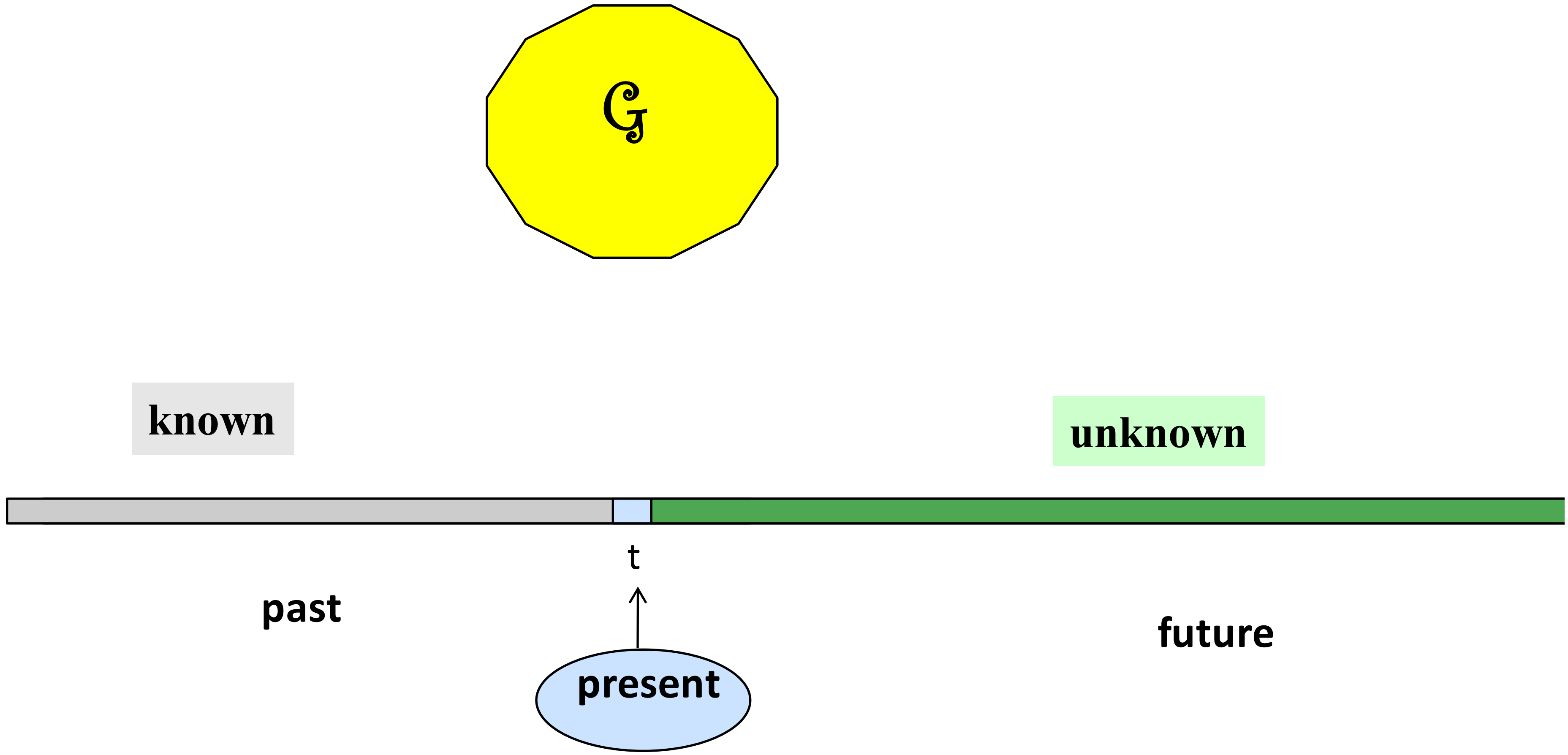
Time-Varying Graph



Time-Varying Graph



Time-Varying Graph

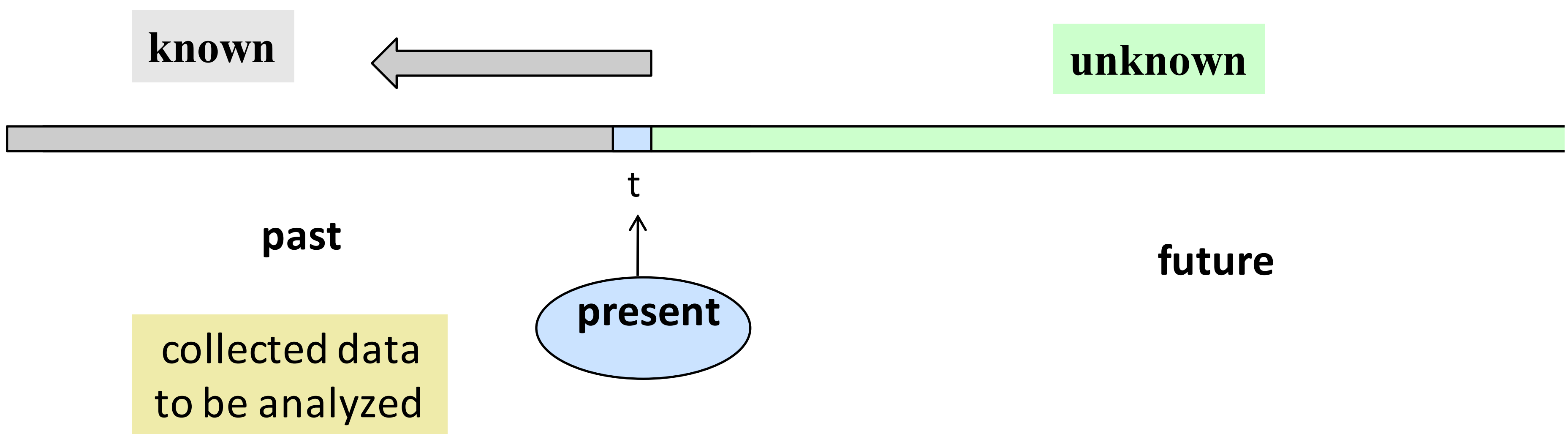


Time-Varying Graph

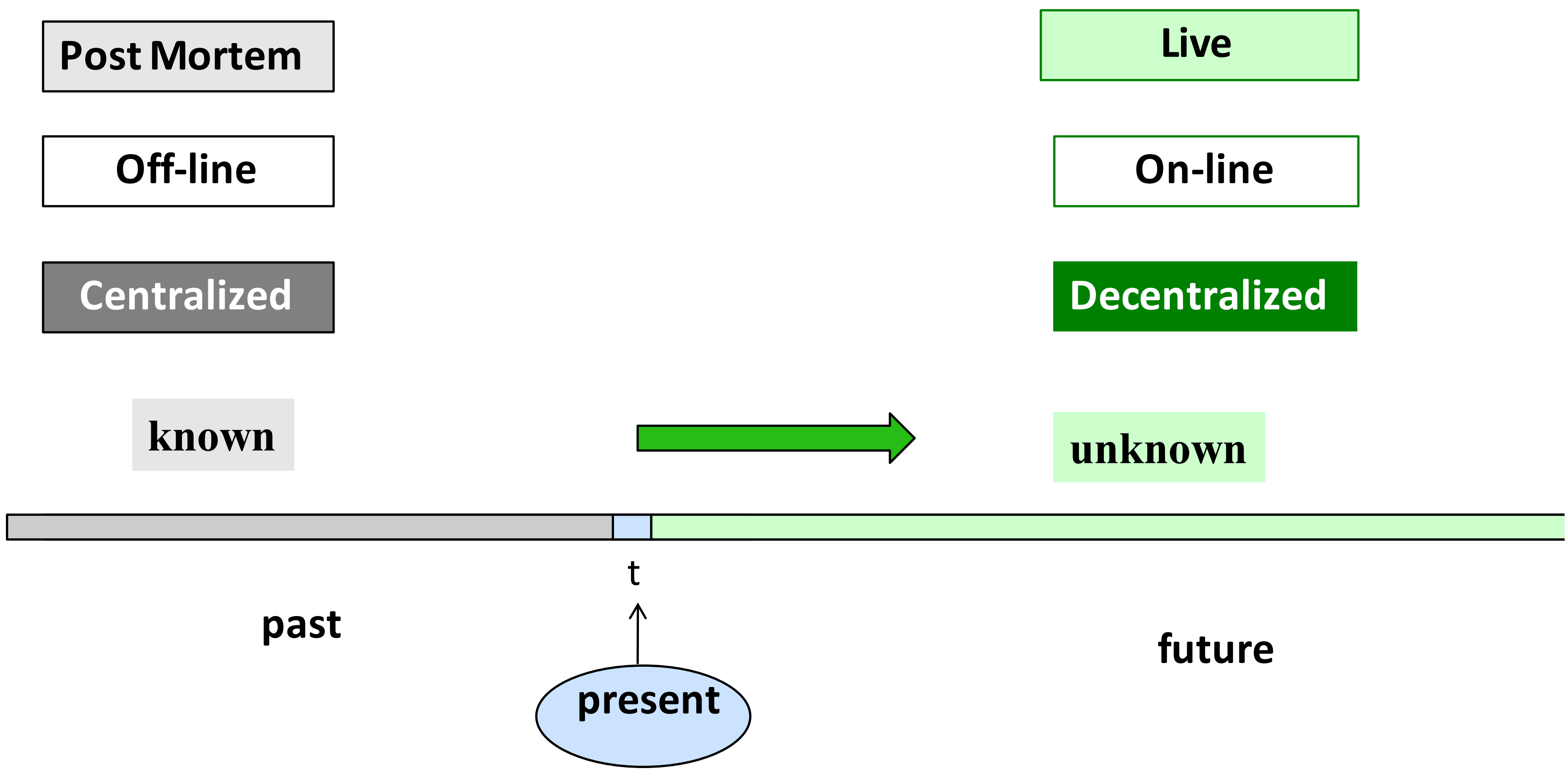
Post Mortem

Off-line

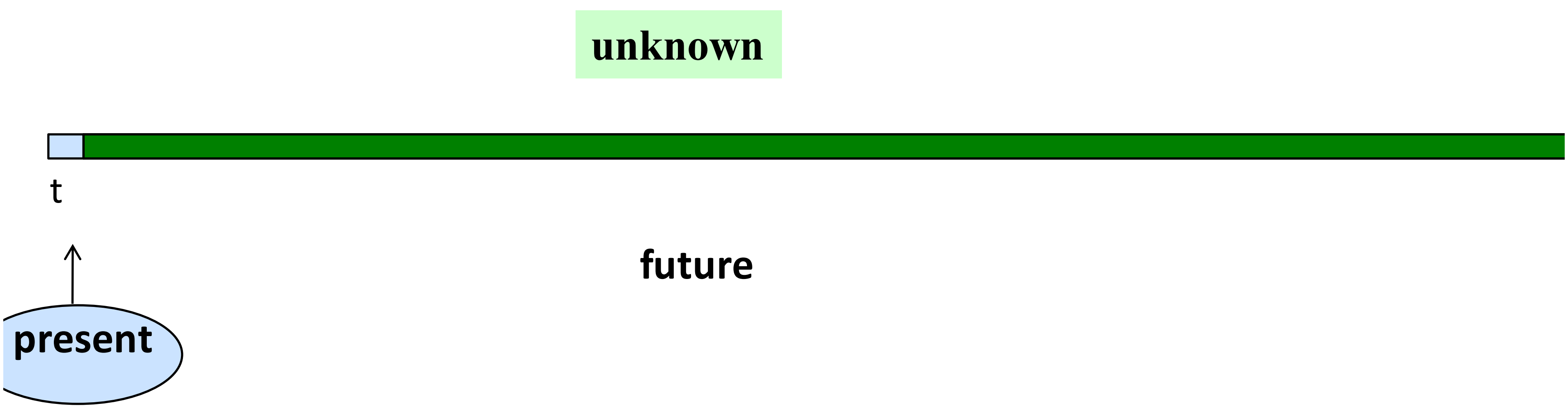
Centralized



Time-Varying Graph

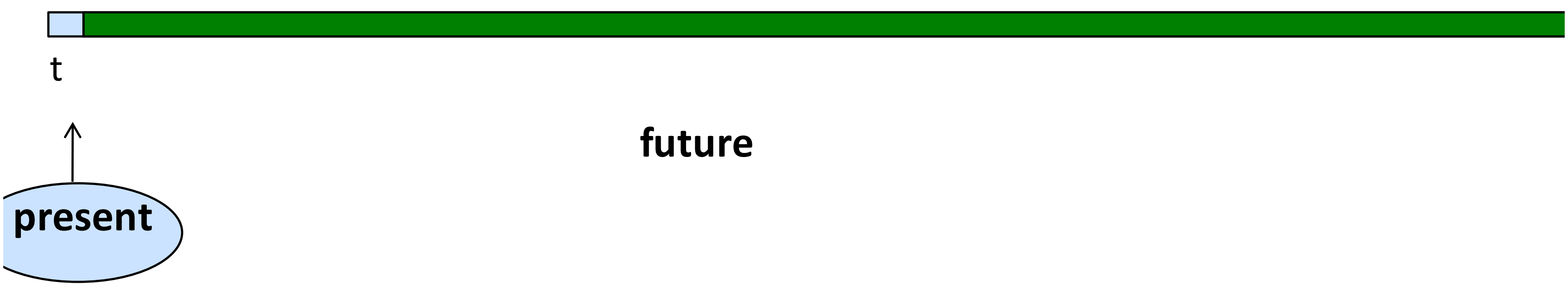


Time-Varying Graph



Time-Varying Graph

something must be known



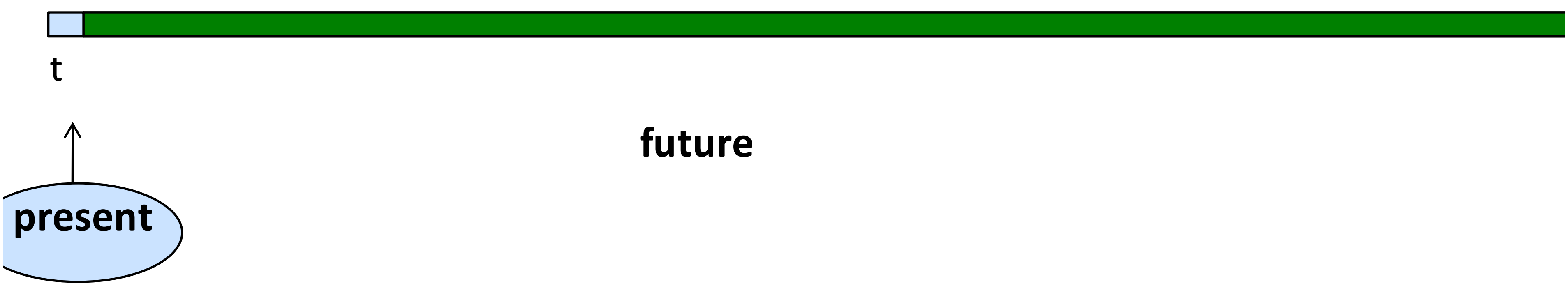
Time-Varying Graph

ASSUMPTIONS

a-priori knowledge

oracle

something must be known



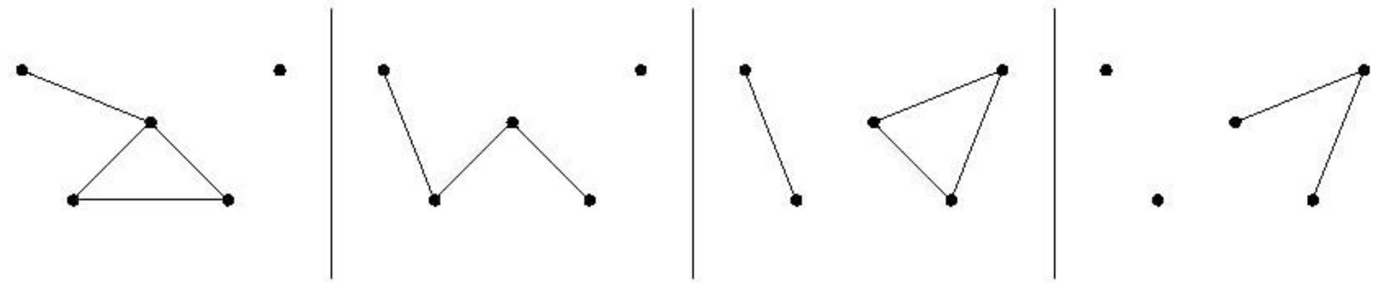
Time-Varying Graph: Common Assumption

FINITE FOOTPRINT G=(N,E)

Time-Varying Graph: Common Assumption

SYNCHRONOUS

Time is divided in rounds

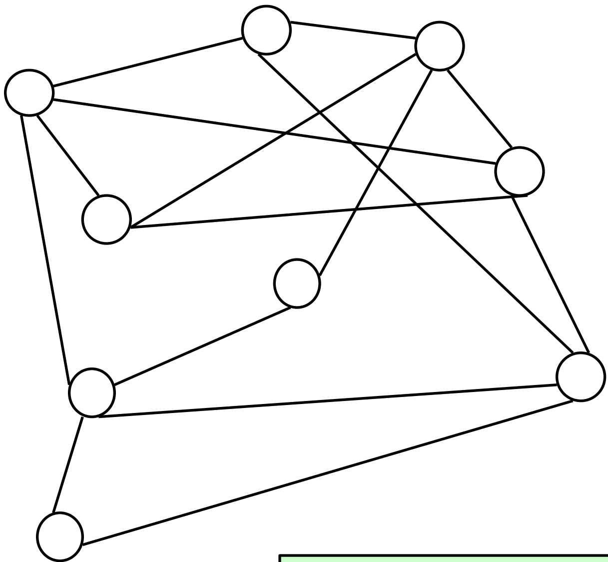


G(0)	G(1)	G(2)	G(3)	...

Evolving graph, Temporal graph, Multi-layer (multiplex)

Dynamic Networks

ASSUMPTIONS



DYNAMICS MODELS
(adversary)

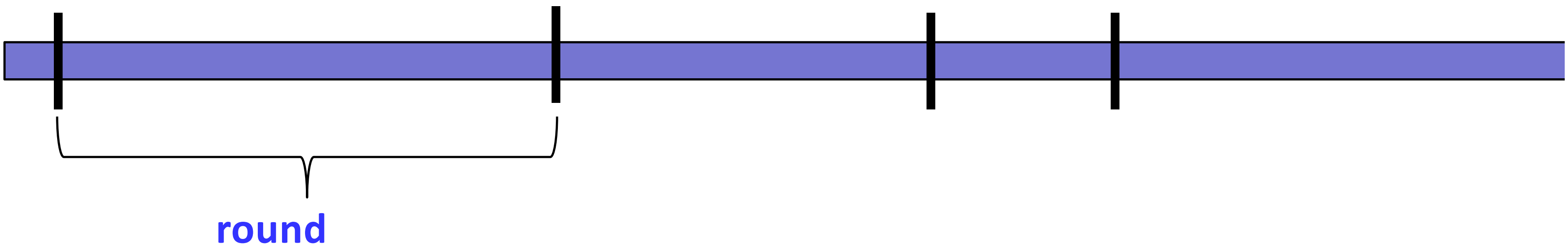
- Temporal Connectivity
- 1-Interval Connectivity
- T-Interval Connectivity

- F. Kuhn, N. Lynch, R. Oshman *STOC* 2010.
- F. Kuhn, Y. Moses, R~Oshman. *PODC* 2011.
- B. Haeuepler, F. Kuhn. *DISC* 2012
- D. Ilcinkas, A.M. Wade. *SIROCCO* 2013
- D. Ilcinkas, R. Klasing, A.M. Wade. *SIROCCO* 2014
- T. Erlerbach, M. Hoffmann, F. Kammer, ICALP 2015
- G.A. Di Luna, S. Dobrev, P. Flocchini, N. Santoro. *ICDCS* 2016

1-Interval-Connectivity

SYNCHRONOUS

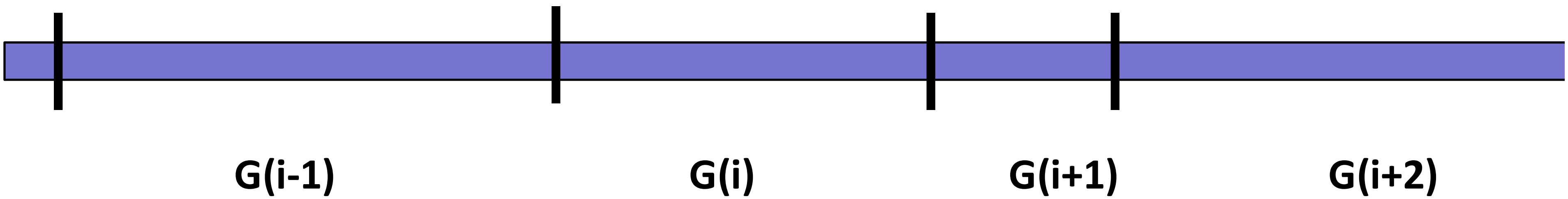
Time is divided in rounds



1-Interval-Connectivity

1-INTERVAL CONNECTED

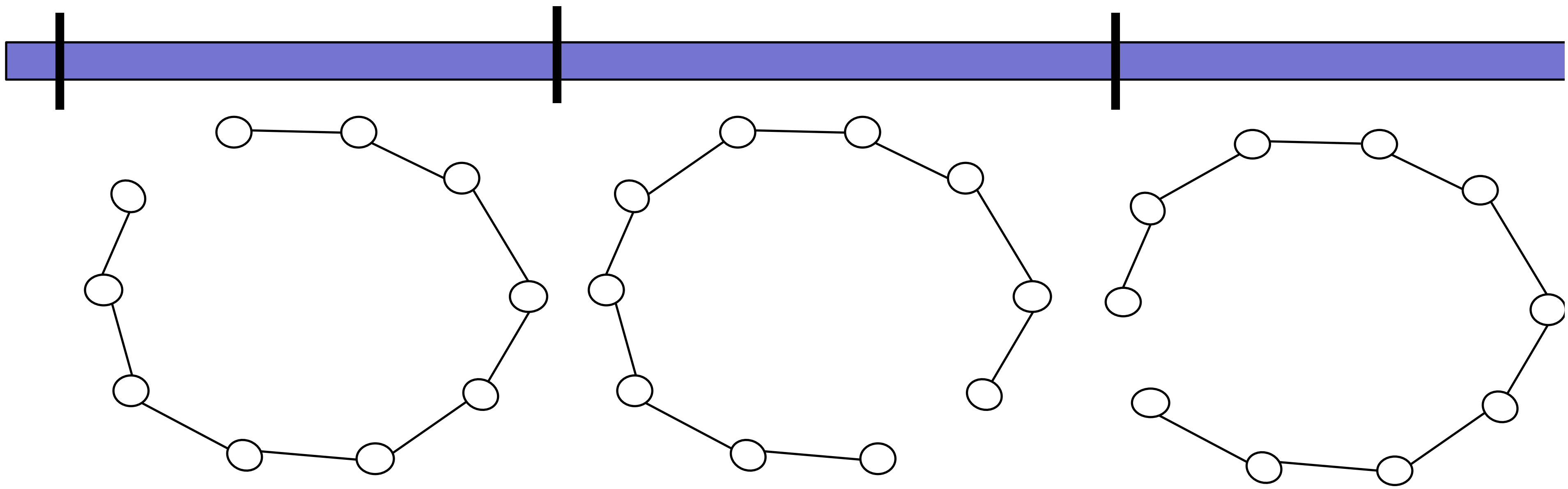
Each G(i) contains a spanning-tree SPT(i) of G



1-Interval-Connectivity

1-INTERVAL CONNECTED

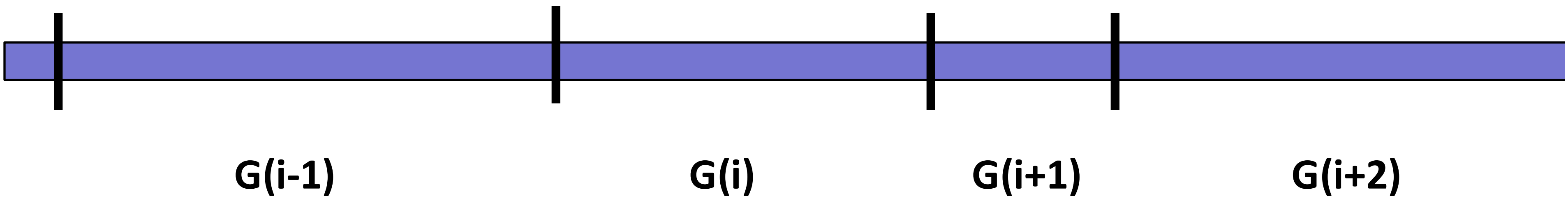
Each G(i) contains a spanning-tree of G



T-Interval-Connectivity

T-INTERVAL CONNECTED

Each G(i) contains a spanning-tree SPT(i) of G

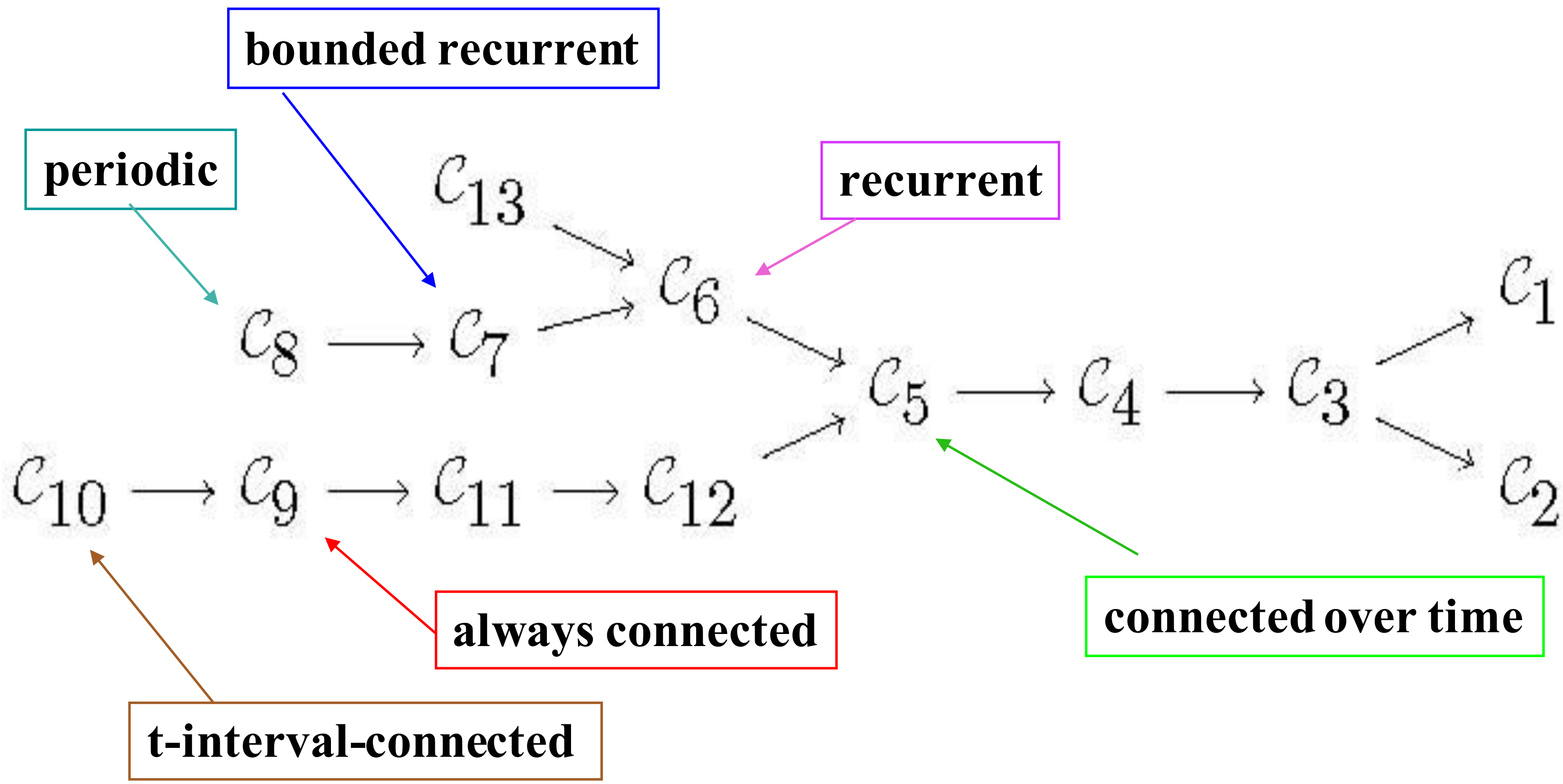


SPT(i) persists for T rounds: i, i+1, i+2, …, i+T-1

Time-Varying Graph

[Casteigts, et al. 2012]

TVG CLASSES



Dynamic Networks : Algorithmic Results

- B.M.B. Xuan, A. F Ferreira, A. Jerry. “Computing shortest, fastest, and foremost Journeys in dynamic networks”. *Int. J. Found. Comput. Sci.* 2003
- Kossinets, Kleinberg, Watts, “The structure of information pathways in a social communication network”. KDD 2008.
- A. Casteigts, P. Flocchini, B.Mans, N. Santoro. “Measuring temporal lags in delay-tolerant networks”. *IEEE Transactions on Computers,* 2014
- A. Casteigts, P. Flocchini, B. Mans, N. Santoro. “Shortest, fastest, and foremost broadcast in dynamic networks”. *Int. J. Foundations of Computer Science*, 2015
- R. O’ Dell and R. Wattenhofer. “Information dissemination in highly dynamic graphs”. DIALM-POMC 2005
- A.Casteigts, S.Chaumette, A.Ferreira. “Characterizing Topological Assumptions of Distributed Algorithms in Dynamic Networks”, SIROCCO 2010
- F. Kuhn, N. Lynch, and R. Oshman. “Distributed computation in dynamic networks”. STOC 2010.

Dynamic Networks : Algorithmic Results

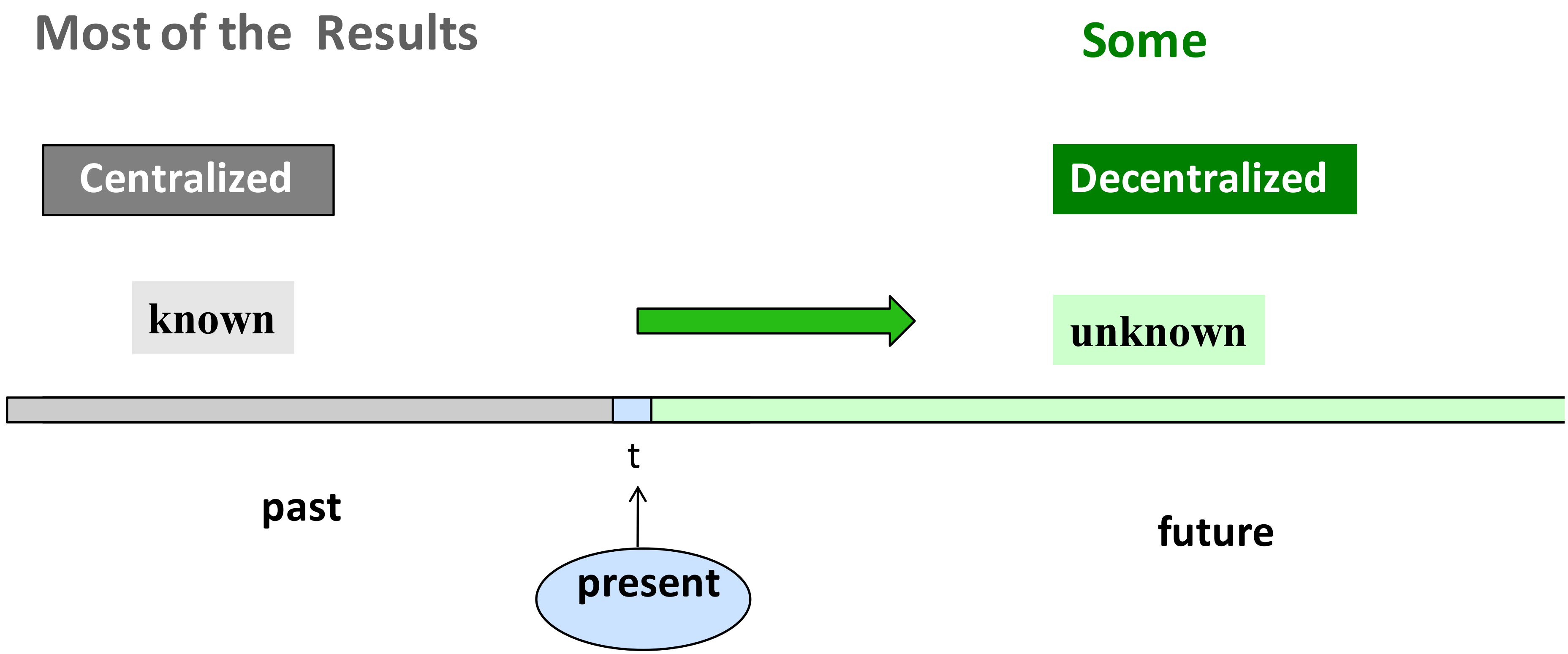
- A. Clementi, F. Pasquale. " Information spreading in dynamic networks". In: *Theoretical Aspects of Distributed Computing in Sensor Networks*, 2011.
- H. Baumann, P. Crescenzi, P. Fraigniaud. "Parsimonious flooding in dynamic graphs". *Distributed Comput*ing, 2011
- F. Kuhn, R. Oshman, Y. Moses. "Coordinated consensus in dynamic networks". PODC 2011.
- B. Haeupler and F. Kuhn. "Lower bounds on information dissemination in dynamic networks". DISC 2012.
- L. Arantes, F. Greve, P. Sens, V. Simon. "Eventual leader election in evolving mobile networks". OPODIS 2013
- J. Augustine, G. Pandurangan, P. Robinson. "Fast Byzantine agreement in dynamic networks. PODC 2013.
- E. Coulouma, E Godard. "A characterization of dynamic networks where consensus is solvable". SIROCCO 2013

Dynamic Networks : Algorithmic Results

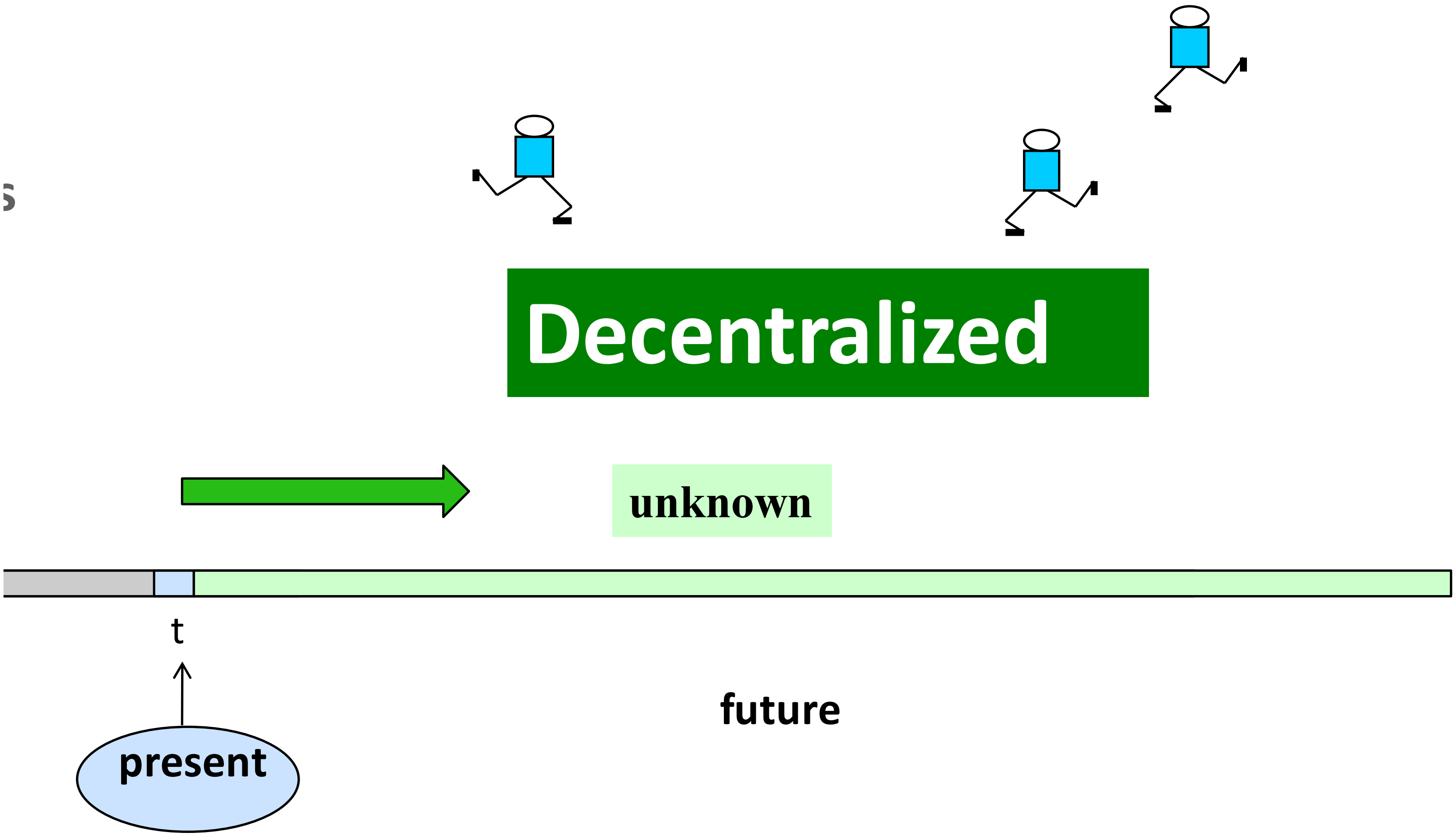
- O. Michail, I. Chatzigiannakis, P. Spirakis. "Naming and counting in anonymous unknown dynamic networks". SSS 2013.
- H. Wu, J. Cheng, S. Huang, Y. Ke, Y. Lu, Y. Xu. "Path problems in temporal graphs". VLDB 2014.
- E.C. Akrida, L. Gasieniec , G.B. Mertzios, P. Spirakis. "Ephemeral networks with random availability of links". SPAA 2014.
- M. Antony, A. Gupta. "Finding a small set of high degree nodes in time-varying graphs". WoWMoM 2014.
- S. Huang, A.W.C. Fu, R Liu. "Minimum spanning trees in temporal graphs. SIGMOD 2015.
- A. Casteigts, R. Klasing, Y.M. Neggaz, J.G. Peters. "Efficiently Testing *T*-Interval Connectivity in Dynamic Graphs". CIAC 2015.
- T. Erlerbach, M. Hoffmann, F. Kammer. "On temporal Graph Exploration", ICALP 2015.

And many more

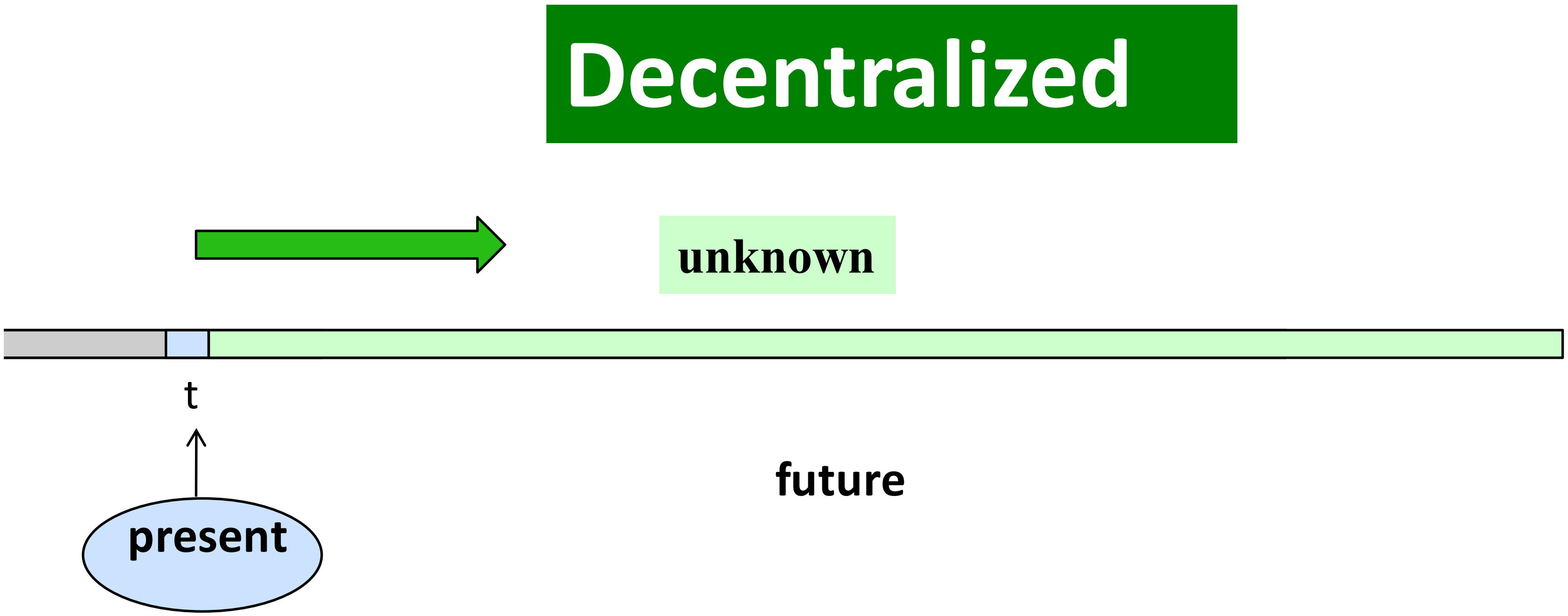
Dynamic Networks : Algorithmic Results



Focus of this talk



Focus of this talk

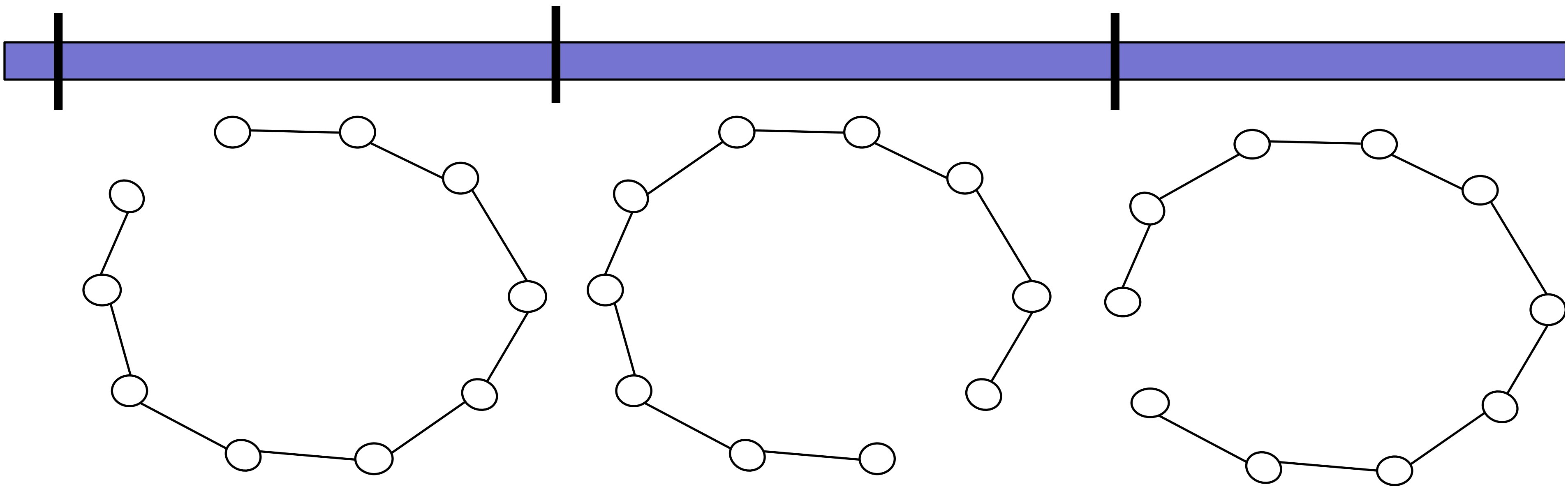


The Model: Dynamic Rings

1-INTERVAL CONNECTED

at each round, the adversary can remove one link

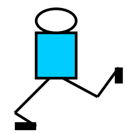
the adversary is possibly **unfair** (a link might be removed forever)



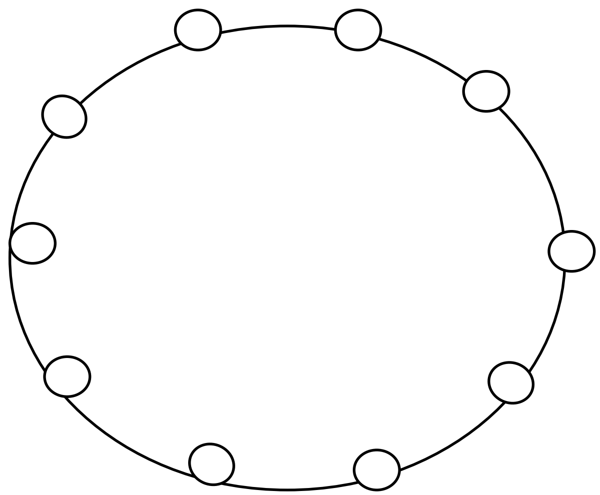
The Model: Dynamic Rings

n nodes

k mobile age**nts**



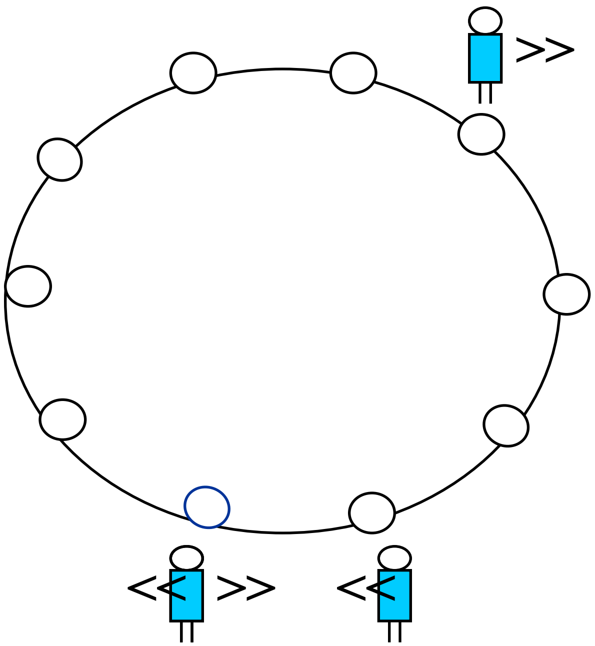
anonymous
silent
bounded memory



The Model: Dynamic Rings

n nodes

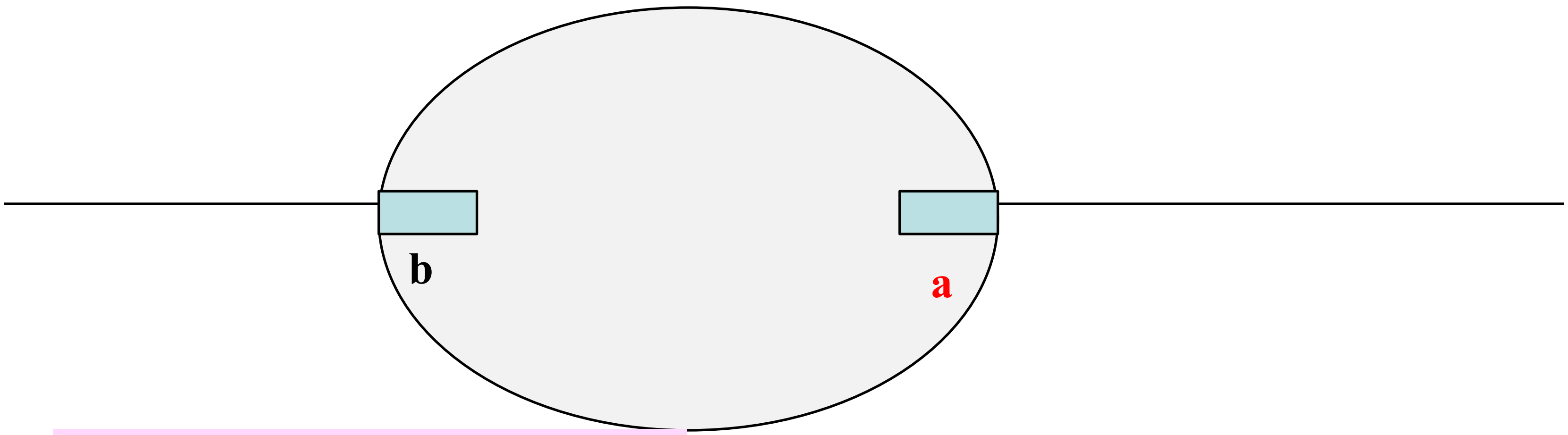
k mobile age**nts**



anonymous
silent
bounded memory
local orientation

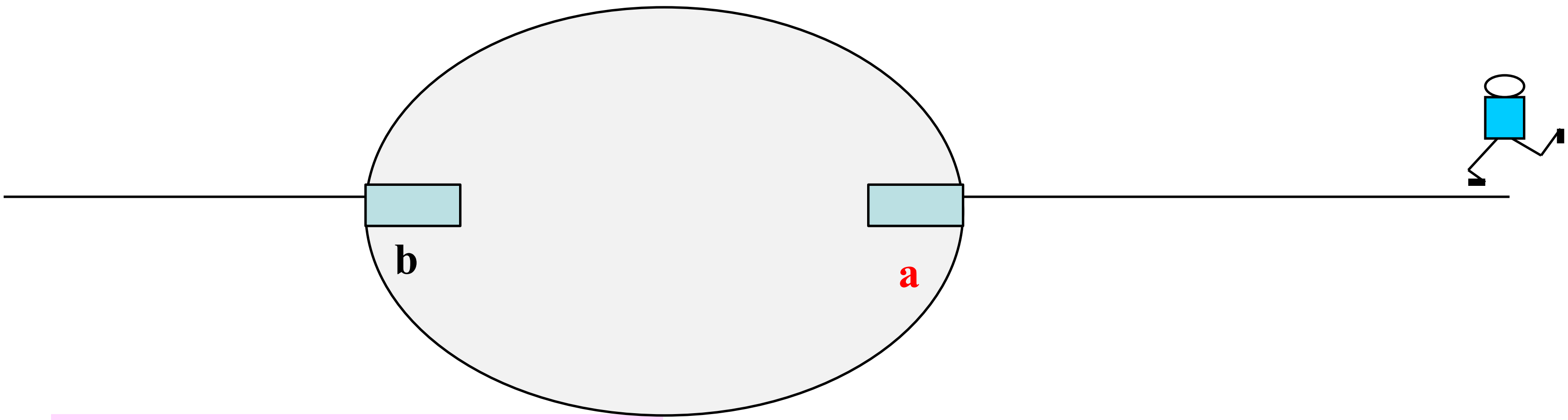
CHIRALITY

The Model: Dynamic Rings



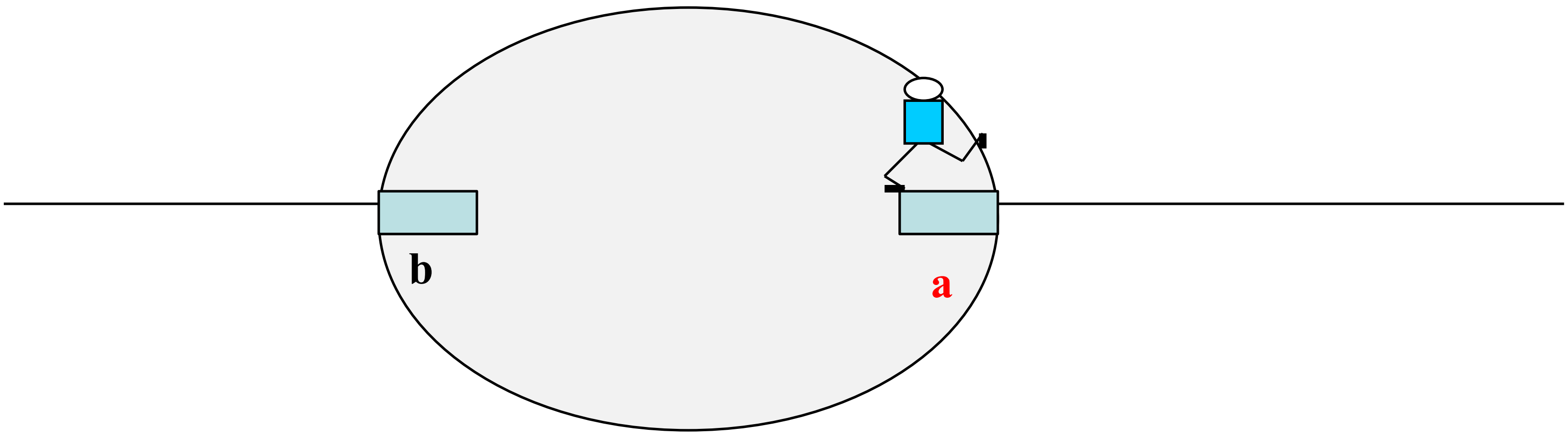
Each node has two distinct **ports**

The Model: Dynamic Rings



When an agent arrives,
it arrives at a port

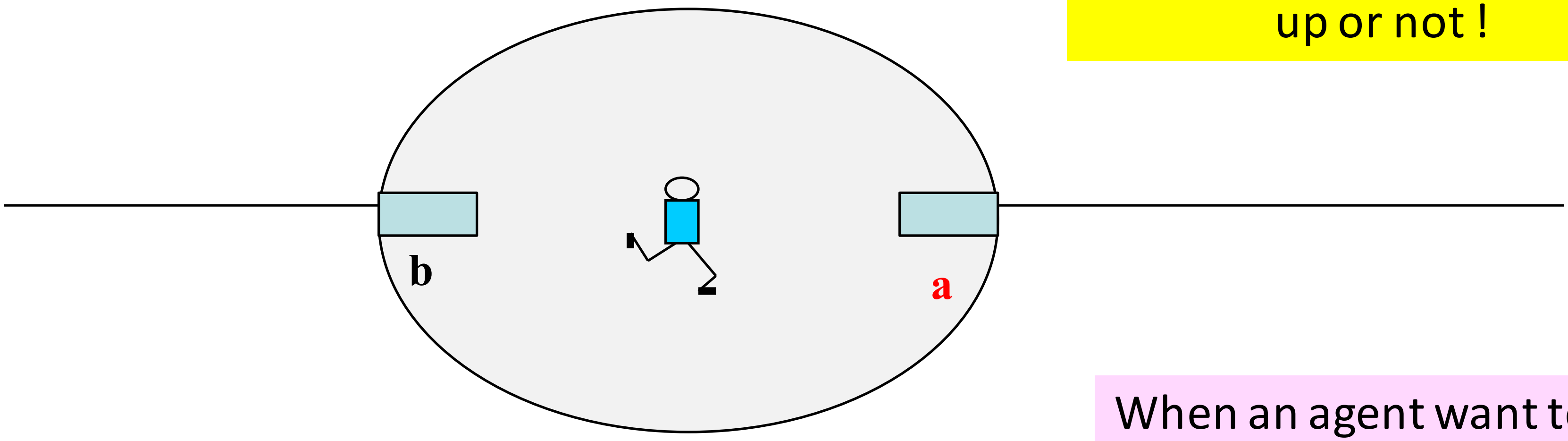
The Model: Dynamic Rings



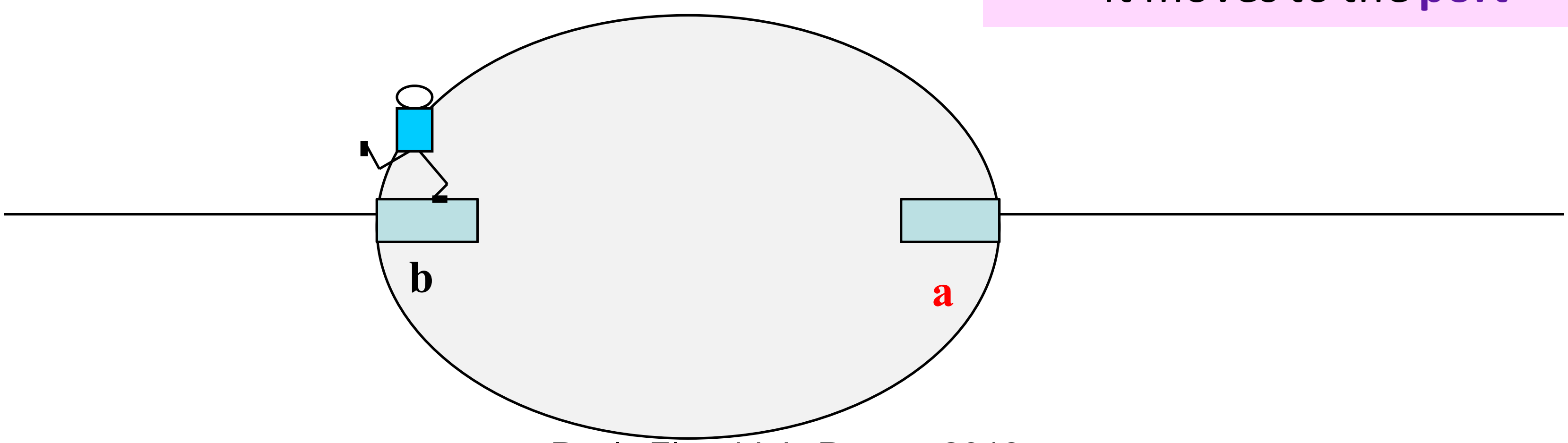
If it decides not to move
(e.g. wait), it goes in the center

The Model: Dynamic Rings

It does not know if the link is up or not !

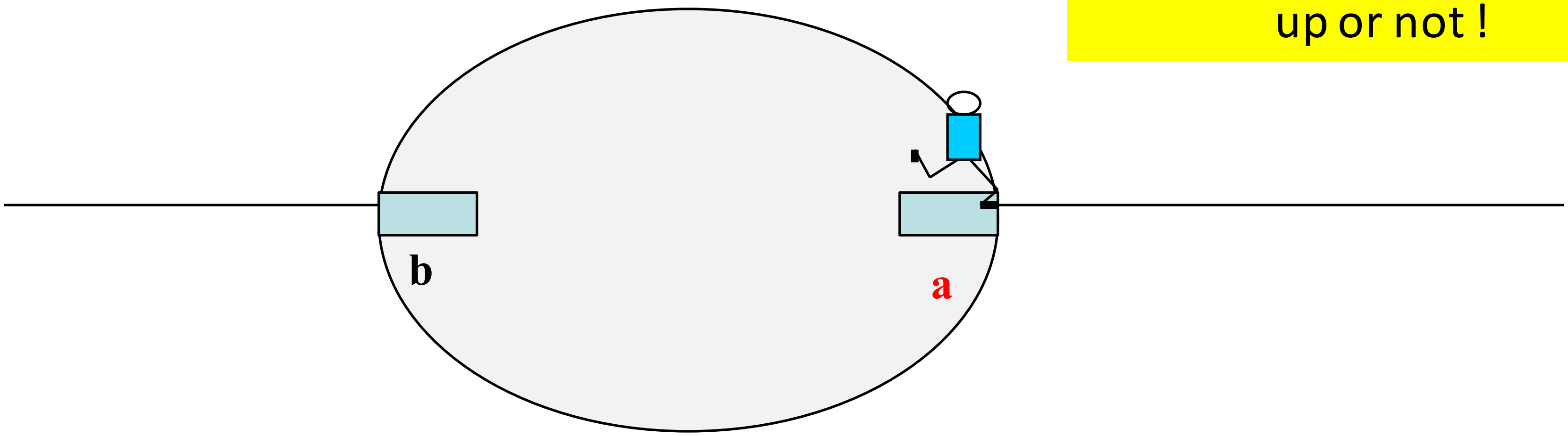


When an agent want to leave it moves to the **port**



The Model: Dynamic Rings

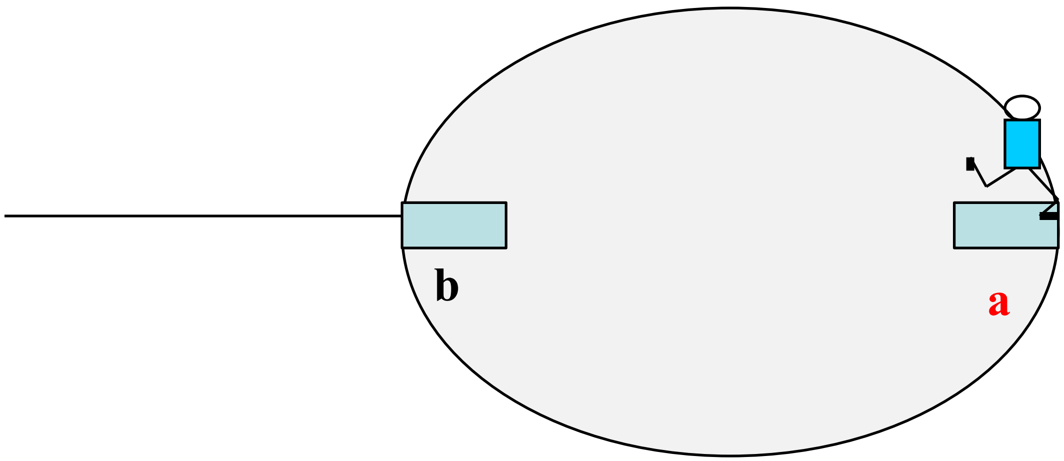
It does not know if the link is up or not !



If the link is there, it arrives at the incident node in the next round

The Model: Dynamic Rings

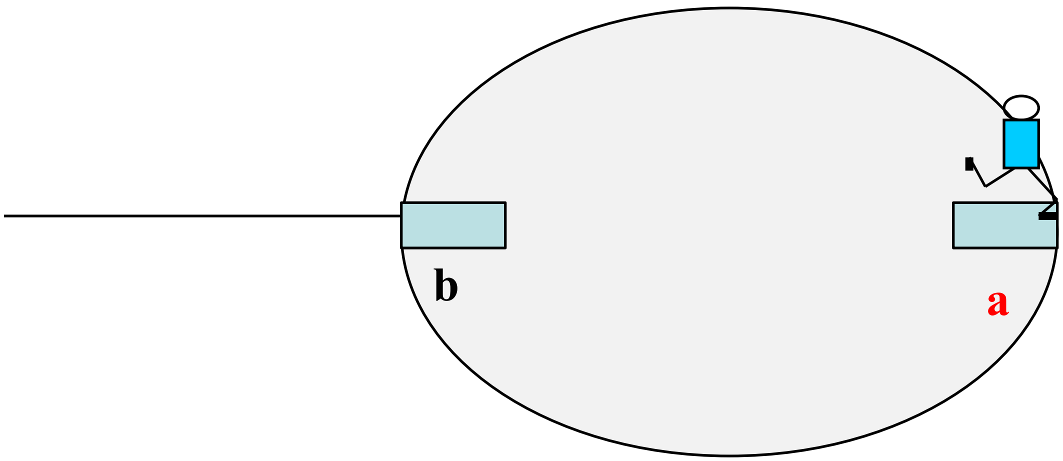
It does not know if the link is up or not !



what if the edge is missing ?

The Model: Dynamic Rings

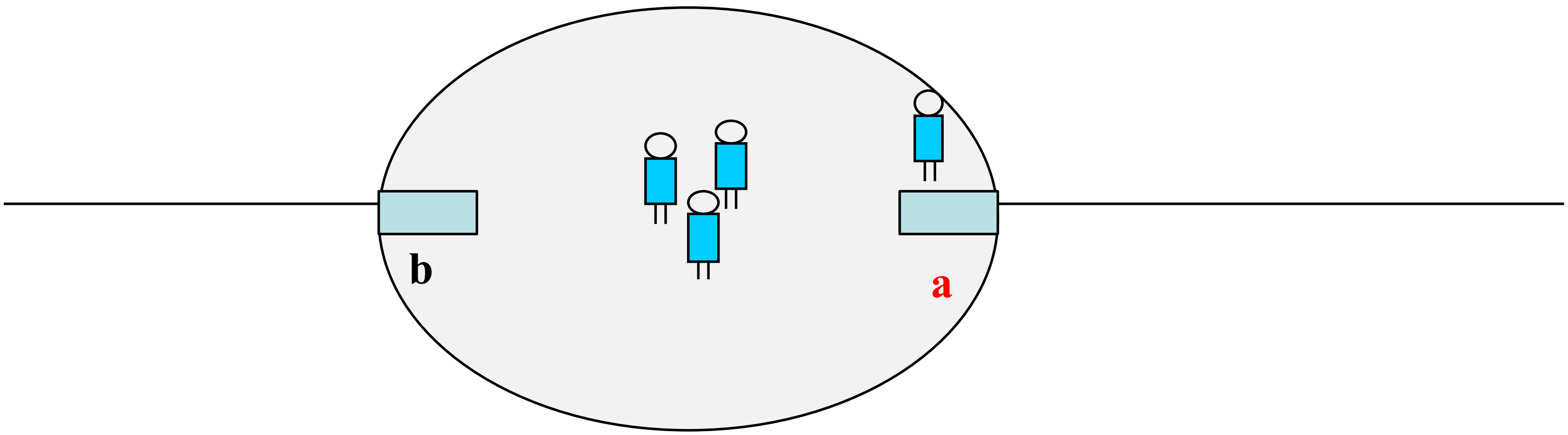
It does not know if the link is up or not !



If the link is missing, it stays on the port until the next round

The Model: Dynamic Rings

FSYNC: all robots are activated in each round

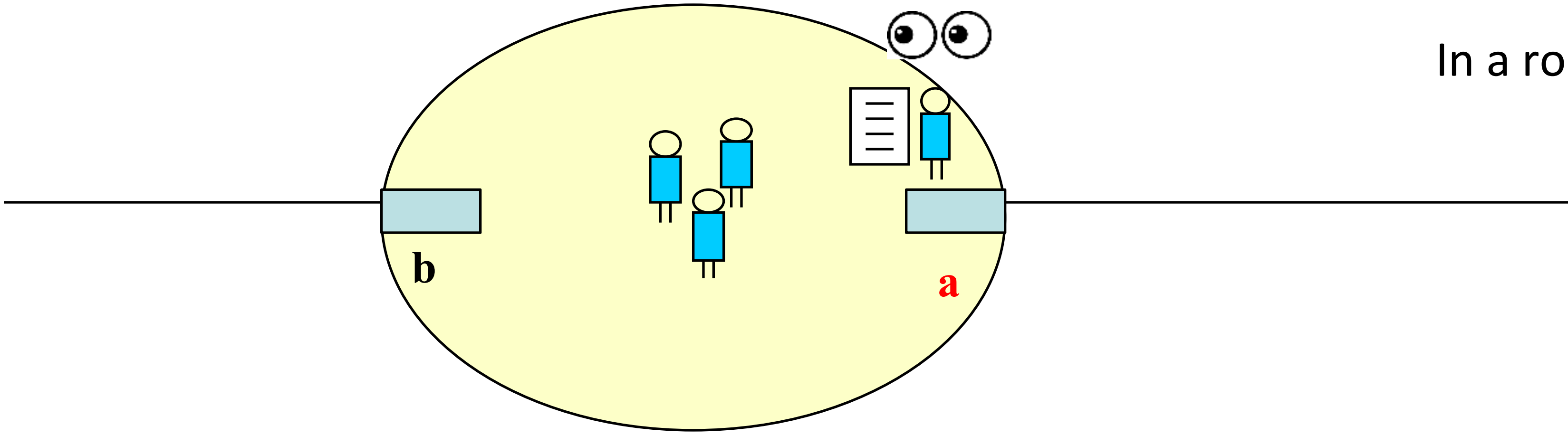


LOOK-COMPUTE-MOVE

No communication (the agents are silent) !!!

The Model: Dynamic Rings

FSYNC: all robots are activated in each round



In a round

LOOK-COMPUTE-MOVE

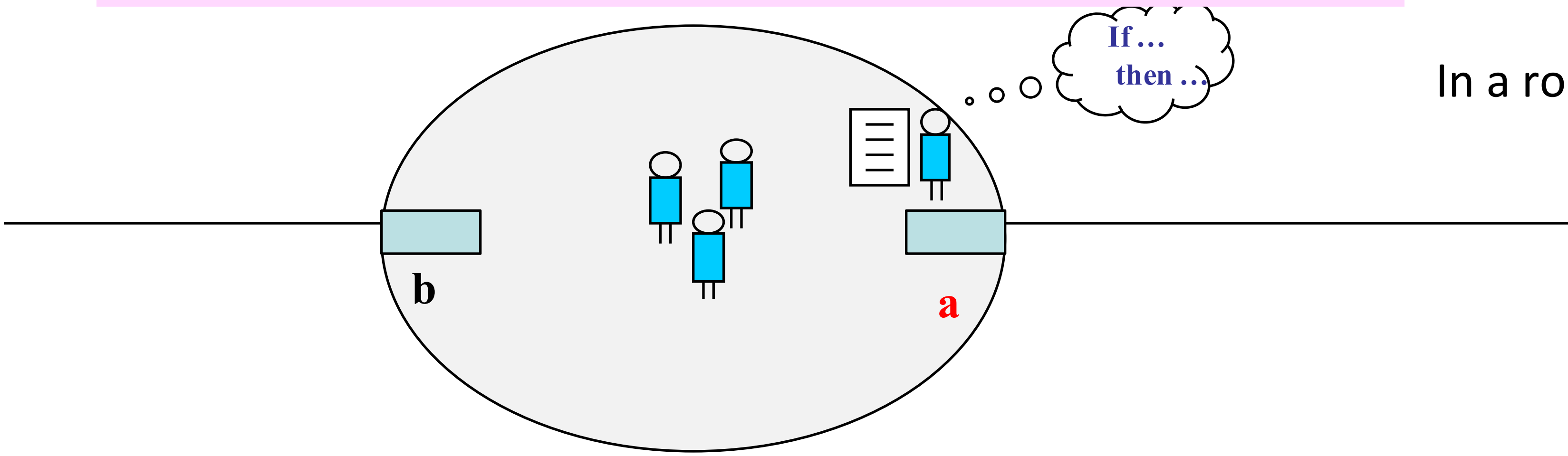
See agents present at the node (center or on ports) and content of memory

Decide what to do (execute algorithm)

Possibly Move

The Model: Dynamic Rings

FSYNC: all robots are activated in each round



In a round

LOOK-**COMPUTE**-MOVE

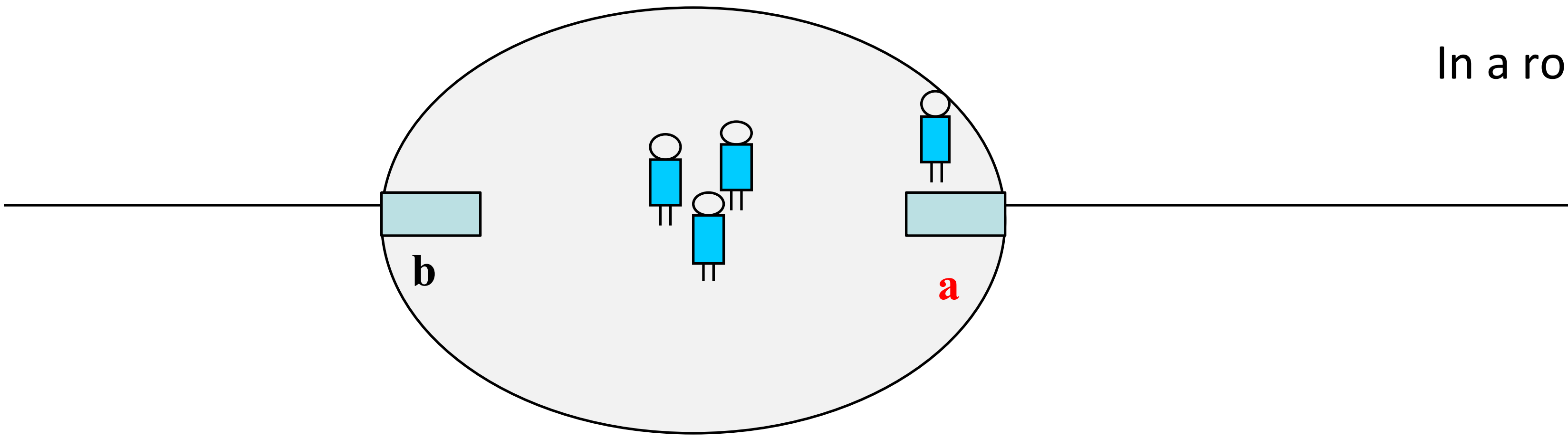
See agents present at the node (center or on ports) and content of memory

Decide what to do (execute algorithm)

Possibly Move

The Model: Dynamic Rings

FSYNC: all robots are activated in each round



In a round

LOOK-COMPUTE-**MOVE**

See agents present at the node (center or on ports) and content of memory

Decide what to do (execute algorithm)

Possibly Move

Mobile Agents in Time-Varying Graphs

RENDEZVOUS/GATHERING

Has been studied only in **STATIC** graphs, and especially in the ring

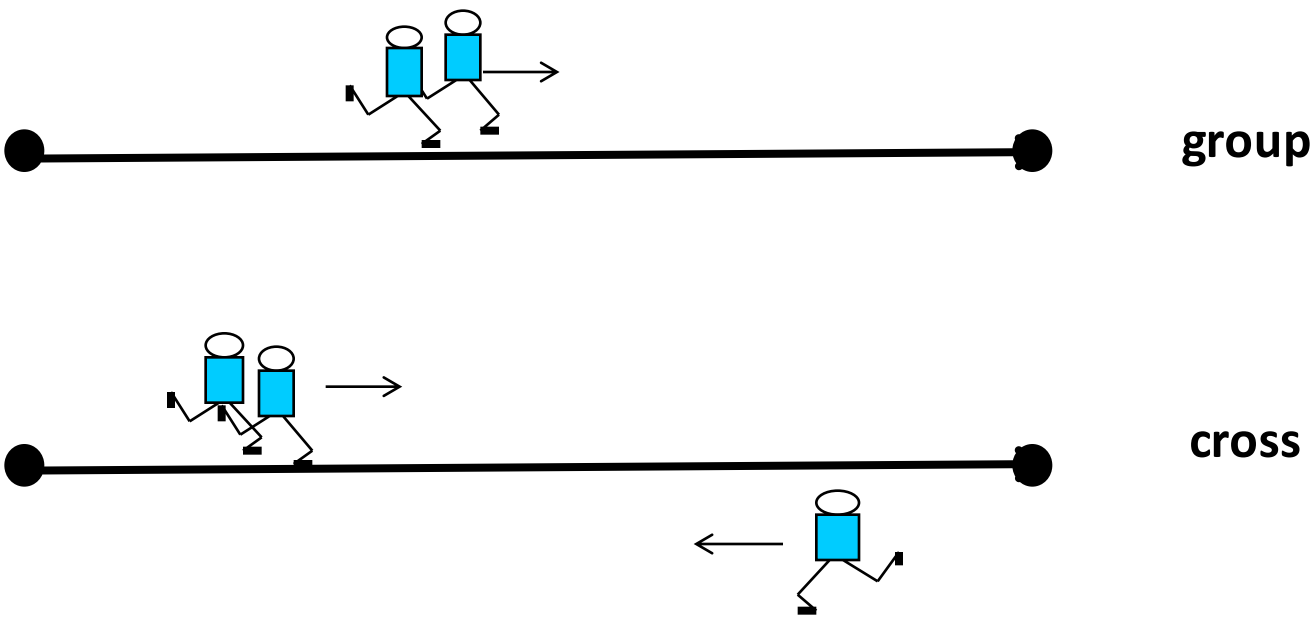
E. Kranakis, D. Krizanc, E. Marcou
The Mobile Agent Rendezvous Problem in the Ring
Morgan & Claypool, 2010

Mobile Agents in Time-Varying Graphs

RENDEZVOUS/GATHERING

G.A. Di Luna, P. Flocchini, G. Prencipe, L. Pagli, N. Santoro, G. Viglietta. "Gathering in dynamic rings". SIROCCO2017.

Gathering in Dynamic Rings



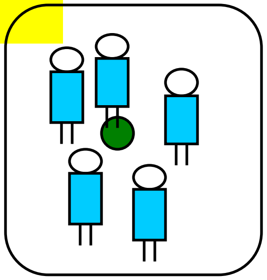
CROSS DETECTION

GATHERING: BASIC LIMITATIONS

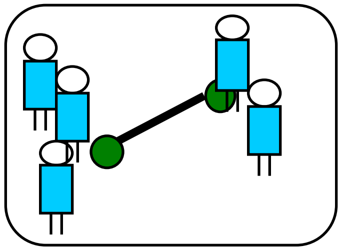
Because of dynamics

T1

Strict Gathering is **unsolvable** in (R, A); this holds regardless of chirality, cross detection, and knowledge of k and n.



→ Strict/Near Gathering



GATHERING: BASIC LIMITATIONS

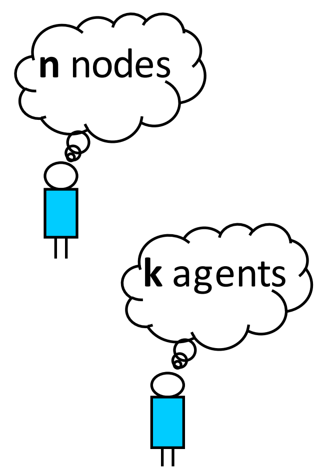
Because of dynamics

T1
Strict Gathering is **unsolvable** in (R, A); this holds regardless of chirality, cross detection, and knowledge of k and n.

Even without dynamics

T2
Gathering is **unsolvable** in (R, A) if neither k nor n are known.

→ n and/or k must be known



GATHERING: BASIC LIMITATIONS

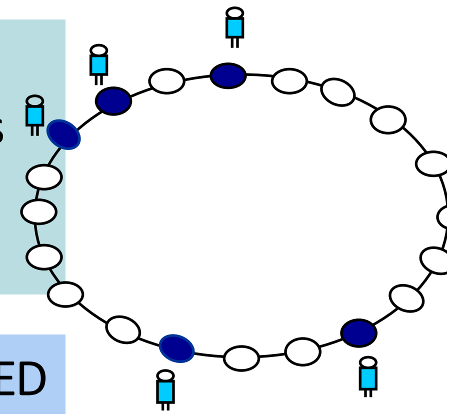
Because of dynamics

T1
Strict Gathering is **unsolvable** in (R, A); this holds regardless of chirality, cross detection, and knowledge of k and n.

Even without dynamics

T2
Gathering is **unsolvable** in (R, A) if neither k nor n are known.

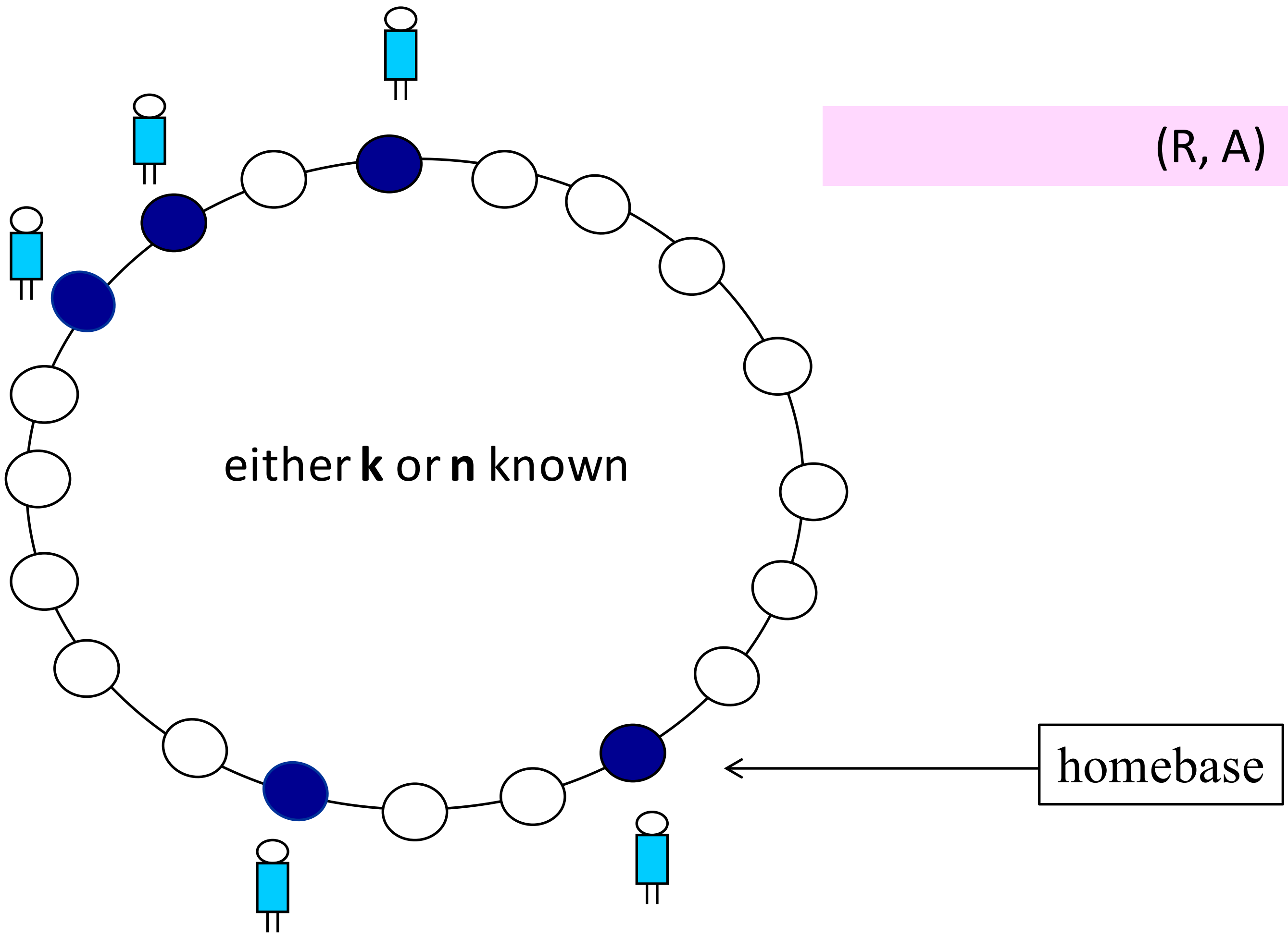
T3
If the homebases are not distinguishable, then Gathering is **unsolvable** in (R, A); this holds regardless of chirality, cross detection, and knowledge of k and n.



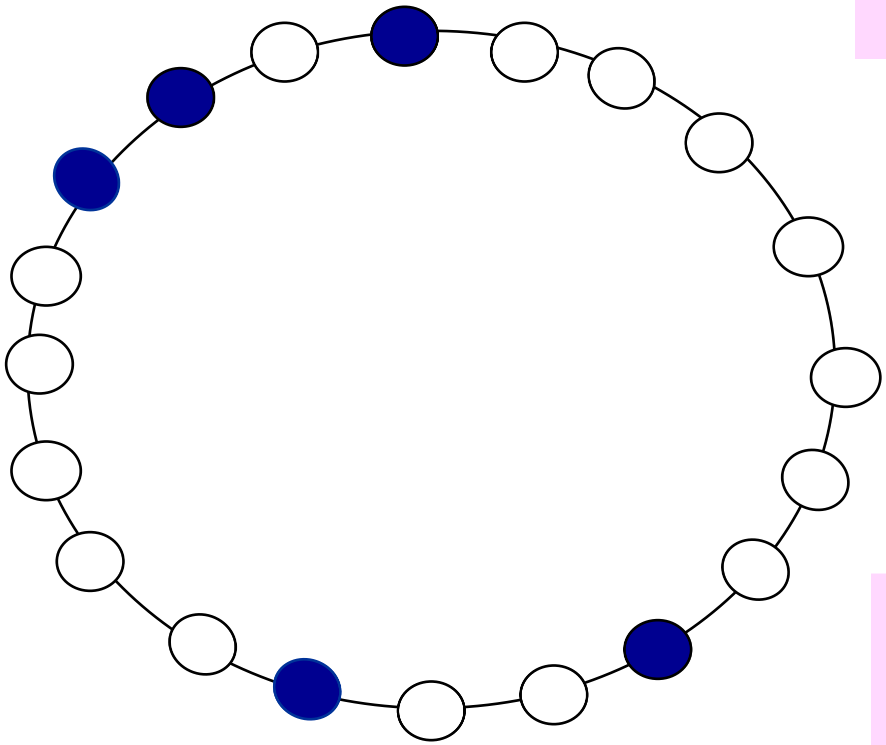
homebases are MARKED

Gathering in Dynamic Rings

(R, A)



Gathering in Dynamic Rings



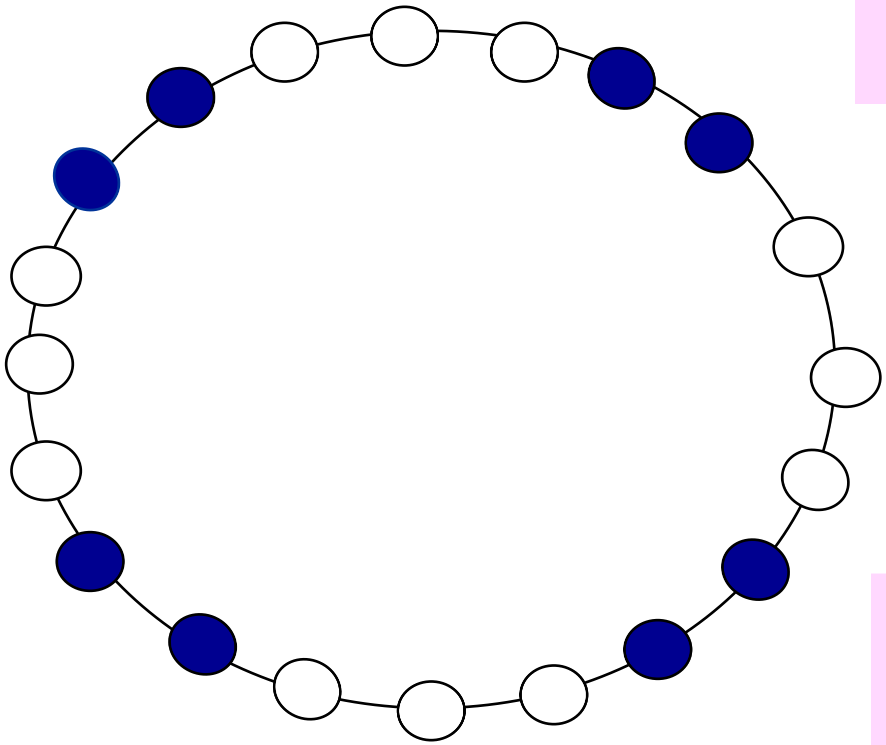
(R, A)

CONFIGURATION

$\mathcal{C}$

Set of all possible configurations

Gathering in Dynamic Rings



$\mathcal{P}$

Set of all periodic configurations

CONFIGURATION

$\mathcal{C}$

Set of all possible configurations

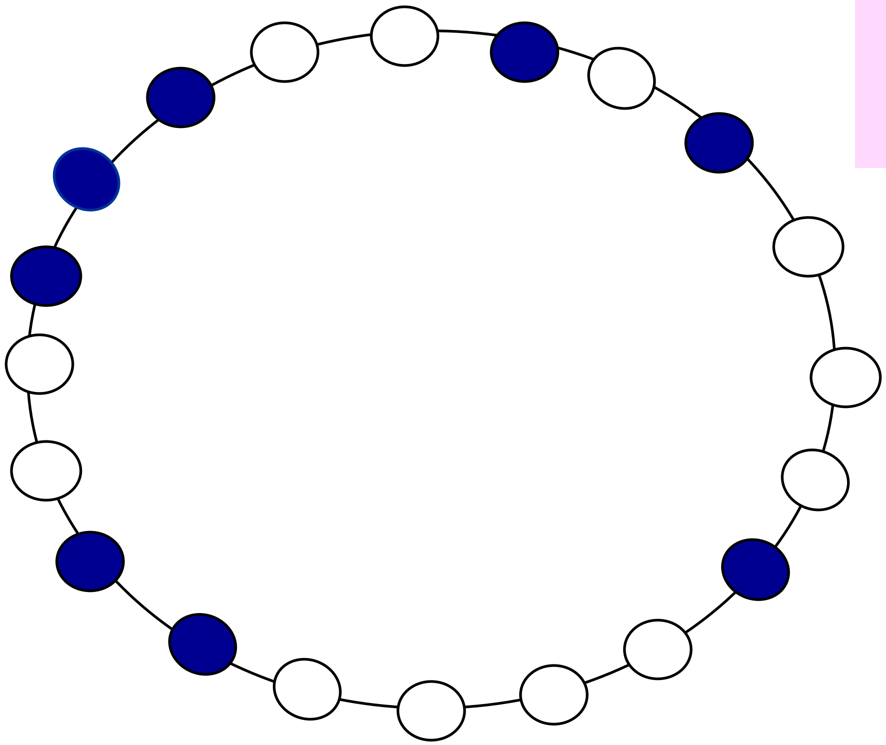
GATHERING: BASIC LIMITATIONS

Even without dynamics

T4
Gathering is **unsolvable** in (R, A) if C ε $\mathcal{P}$; this holds regardless of chirality, cross detection, and knowledge of k and n.

⇒ C is not periodic

Gathering in Dynamic Rings

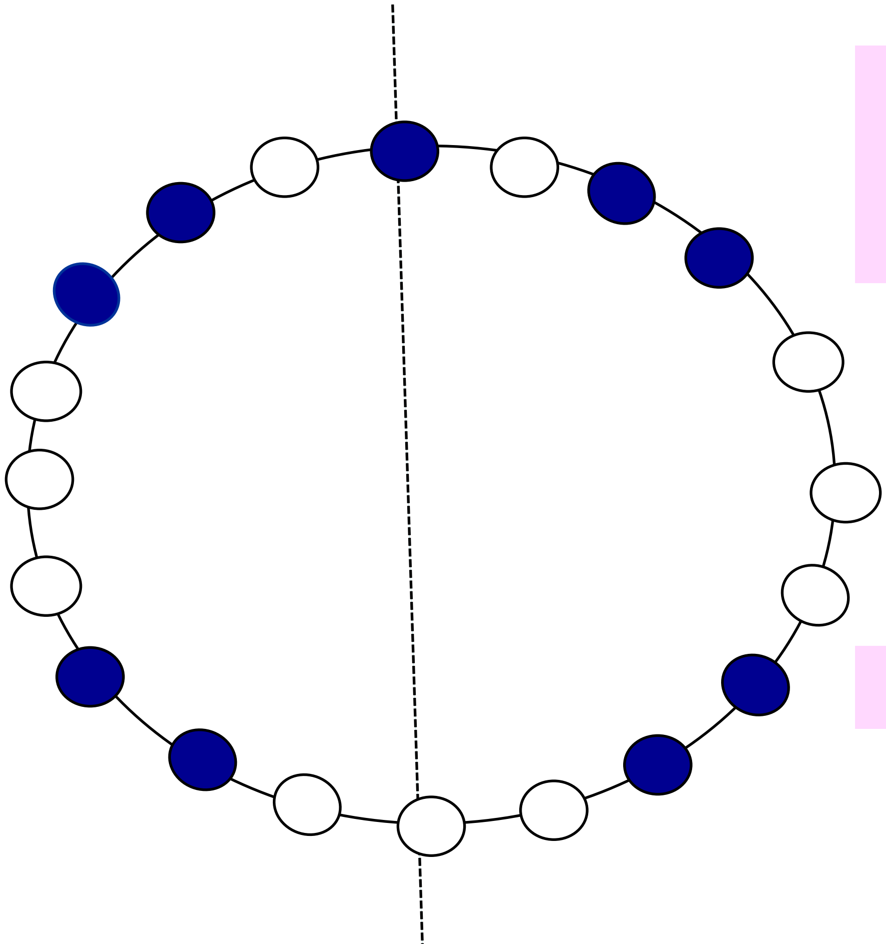


$\mathcal{A}$

Aperiodic configurations
(the only feasible ones in static)

CONFIGURATION

Gathering in Dynamic Rings



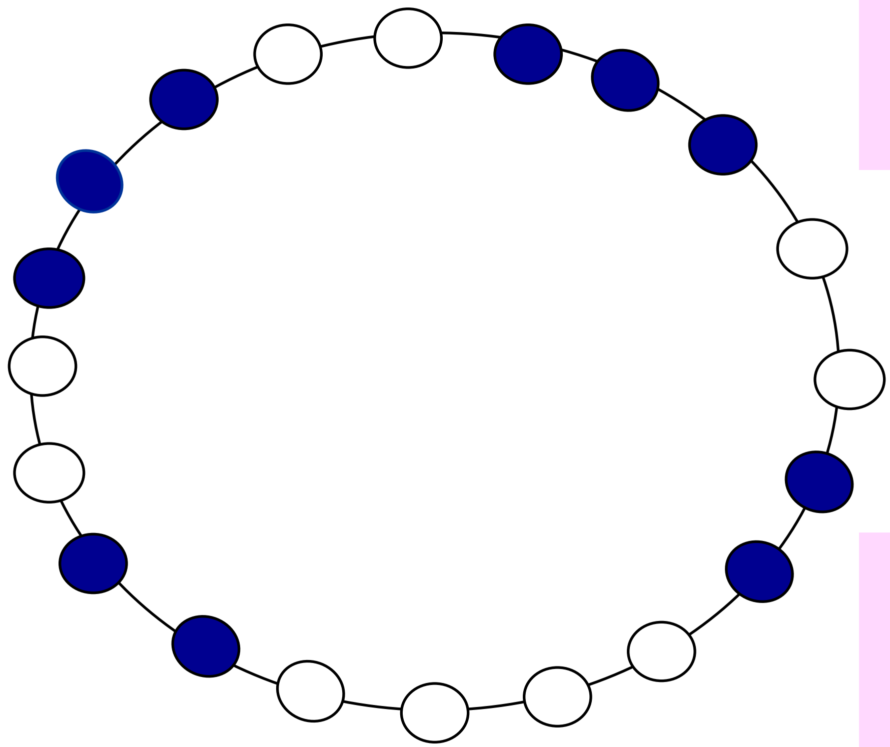
$\mathcal{A}$

Aperiodic configurations
(the only feasible ones in static)

CONFIGURATION

Double palindromes

Gathering in Dynamic Rings



$\mathcal{A}$

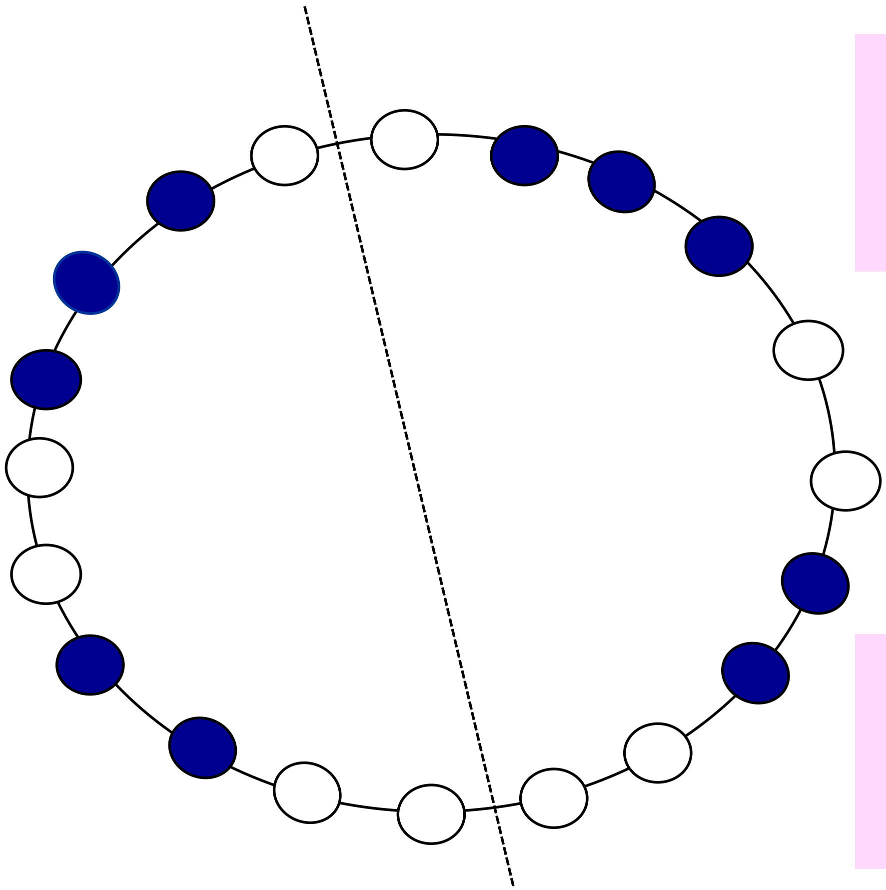
Aperiodic configurations
(the only feasible ones in static)

CONFIGURATION

$\mathcal{E}$

Double palindromes with
edge-edge axis of symmetry

Gathering in Dynamic Rings



$\mathcal{A}$

Aperiodic configurations
(the only feasible ones in static)

CONFIGURATION

$\mathcal{E}$

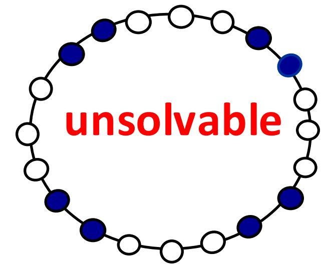
Double palindromes with
edge-edge axis of symmetry

GATHERING: BASIC LIMITATIONS

Even without dynamics

T4

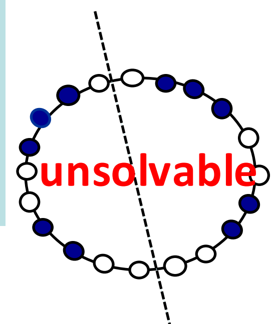
Gathering is **unsolvable** in (R, A) if C ε $\mathcal{P}$; this holds regardless of chirality, cross detection, and knowledge of k and n**.**



Because of dynamics

T5

Without cross-detection and without chirality Gathering is **unsolvable** in (R, A) if C ε $\mathcal{E}$; this holds regardless of knowledge of k and n**.**



GATHERING: FEASIBILITY

	chirality	no chirality
cross detection	$\mathcal{A}$	$\mathcal{A}$
no cross detection	$\mathcal{A}$	$\mathcal{A} \setminus \mathcal{E}$

With knowledge of n

GATHERING: FEASIBILITY

	chirality	no chirality
cross detection	$\mathcal{A}$	$\mathcal{A}$
no cross detection	$\mathcal{A}$	$\mathcal{A} \setminus \mathcal{E}$

With knowledge of n

GATHERING: FEASIBILITY

	chirality	**no chirality**
cross detection	$\mathcal{A}$	$\mathcal{A}$
no cross detection	$\mathcal{A}$	$\mathcal{A} \setminus \mathcal{E}$

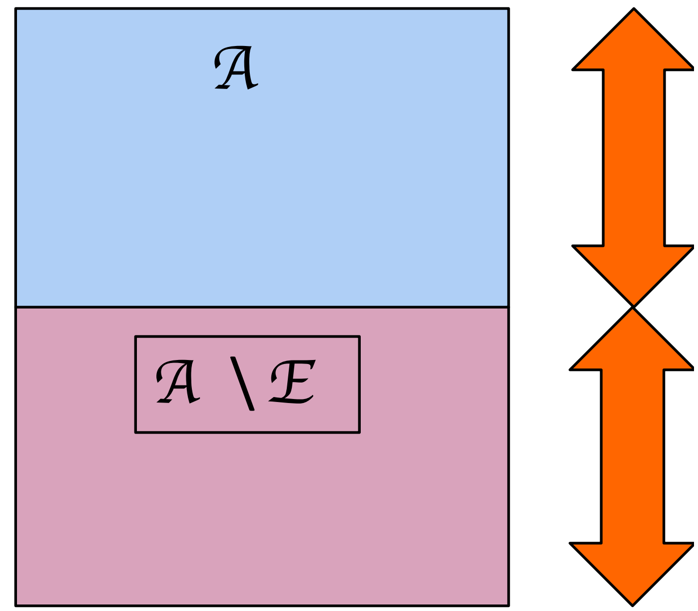
With knowledge of n

GATHERING: FEASIBILITY

no chirality

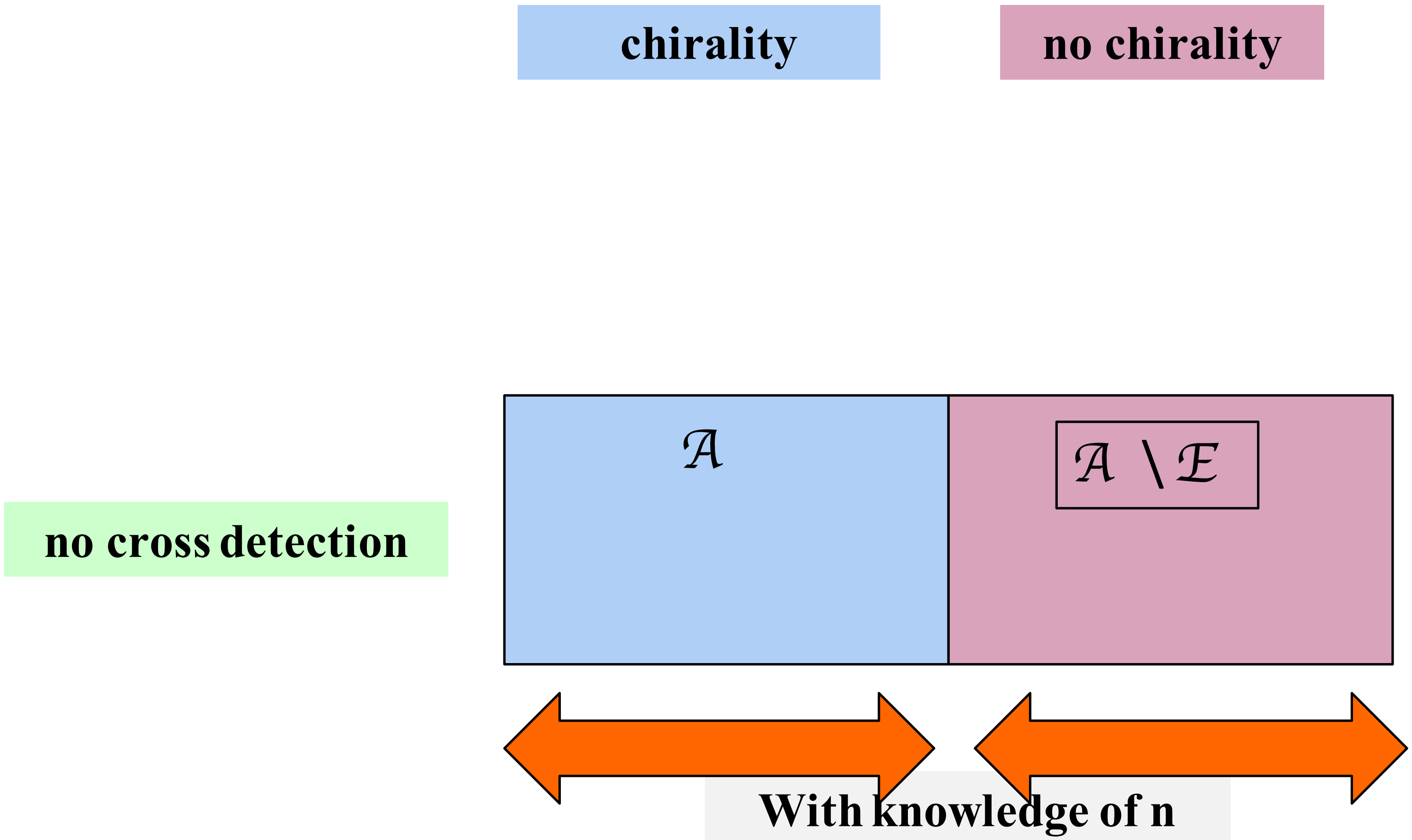
cross detection

no cross detection



With knowledge of n

GATHERING: FEASIBILITY



GATHERING: FEASIBILITY

	chirality	no chirality
cross detection	$\mathcal{A}$	$\mathcal{A} \setminus \mathcal{E}$
no cross detection	$\mathcal{A}$	$\mathcal{A} \setminus \mathcal{E}$

Knowledge of n is more powerful

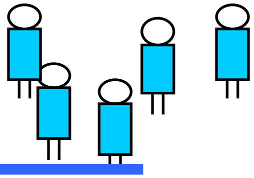
With knowledge of k only

GATHERING: FEASIBILITY

	chirality	no chirality
cross detection	$\mathcal{A}$	$\mathcal{A} \setminus \mathcal{E}$
no cross detection	$\mathcal{A}$	$\mathcal{A} \setminus \mathcal{E}$

With knowledge of k only

GATHERING: GENERAL SOLUTION STRUCTURE

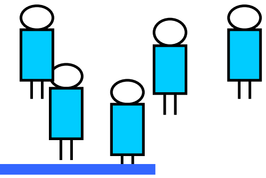


Different strategies depending on

- availability or lack of **cross detection**

- presence or absence of **chirality**

GATHERING: GENERAL SOLUTION STRUCTURE



Two phases

Phase 1: The agents explore the ring

They might already solve Gathering. If so, they stop.

If not, the agents are able to elect a node or an edge and proceed to Phase 2

Phase 2: The agents gather

They try to gather around the elected node or edge.

If that is not possible (due to the ring dynamics), gathering occurs nevertheless at another place.

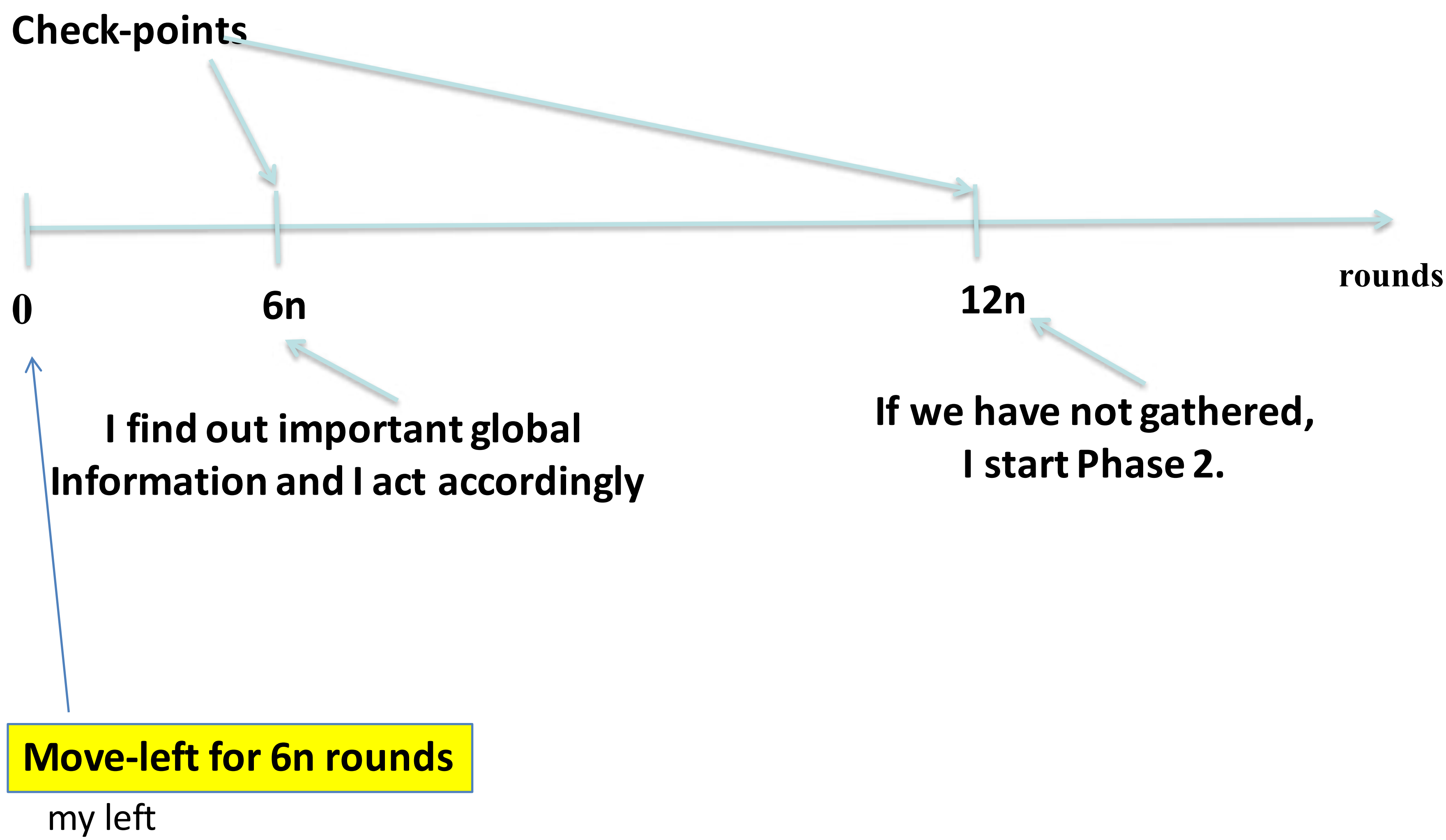
GATHERING: CROSS DETECTION - NO CHIRALITY

	chirality	no chirality
cross detection	$\mathcal{A}$	$\mathcal{A}$
no cross detection	$\mathcal{A}$	$\mathcal{A} \setminus \mathcal{E}$

With knowledge of n

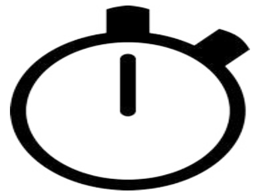
With Cross Detection: Without Chirality

Phase 1: Exploration



With Cross Detection: Without Chirality

Phase 1: Exploration



Special Condition checked at round 6n:

P: last time I met someone new going in my direction was less than 3**n** rounds ago; since then I traversed less than **n** links.

P **true** at round 6n means:

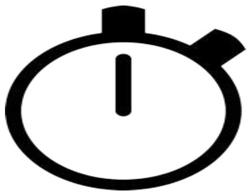
All agents moving in my direction form a single group; some may have not explored the whole ring; **P is true for all of them.**

P **false** at round 6n means:

All agents moving in my direction have explored the whole ring (hence they know k and the configuration)**, and P is false also for them.**

With Cross Detection: Without Chirality

Phase 1: Exploration


round 6n

Special Condition checked at round 6n:

P: last time I met someone new going in my direction was less than 3**n** rounds ago; since then I traversed less than **n** links.

If P is true, I continue in the same direction for 6n more rounds

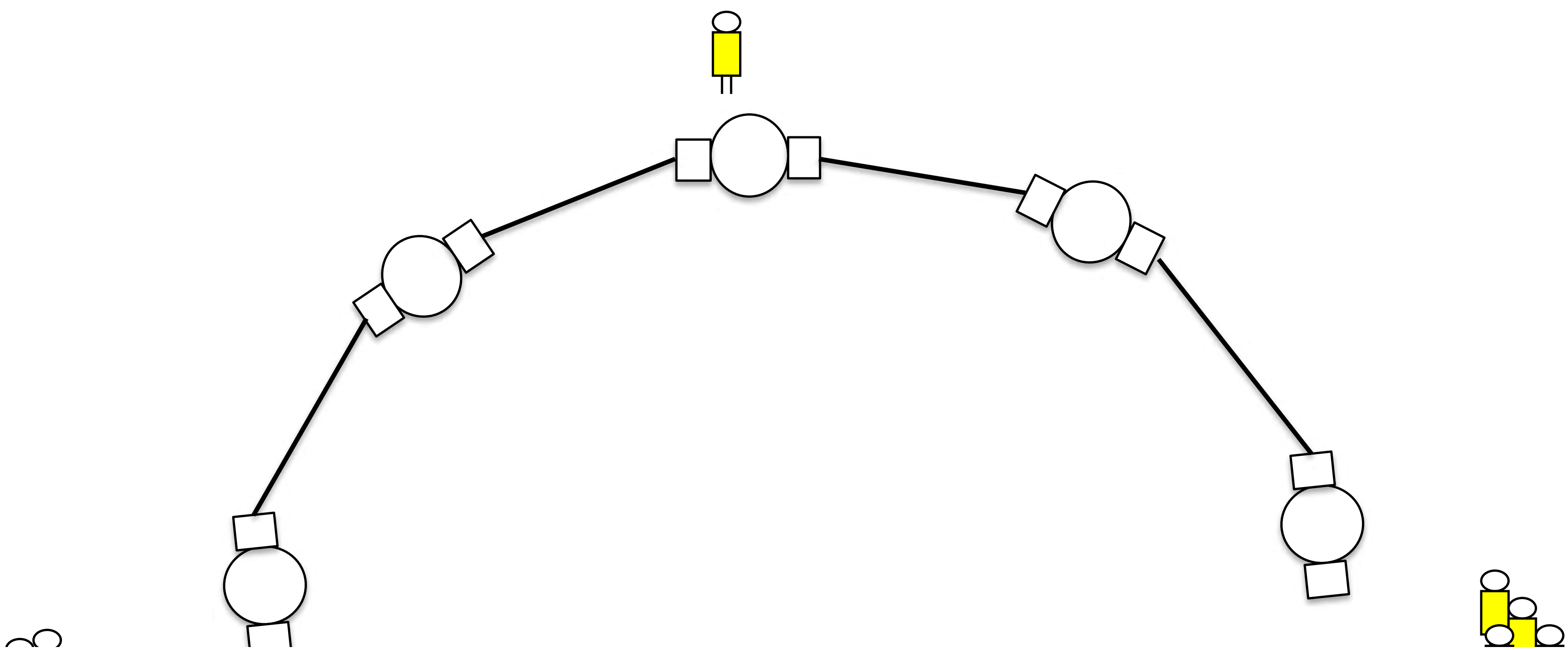
If P is false, I switch direction and move for 6n more rounds

During this time, I may TERMINATE if certain conditions occur

With Cross Detection: Without Chirality

Phase 1: Exploration

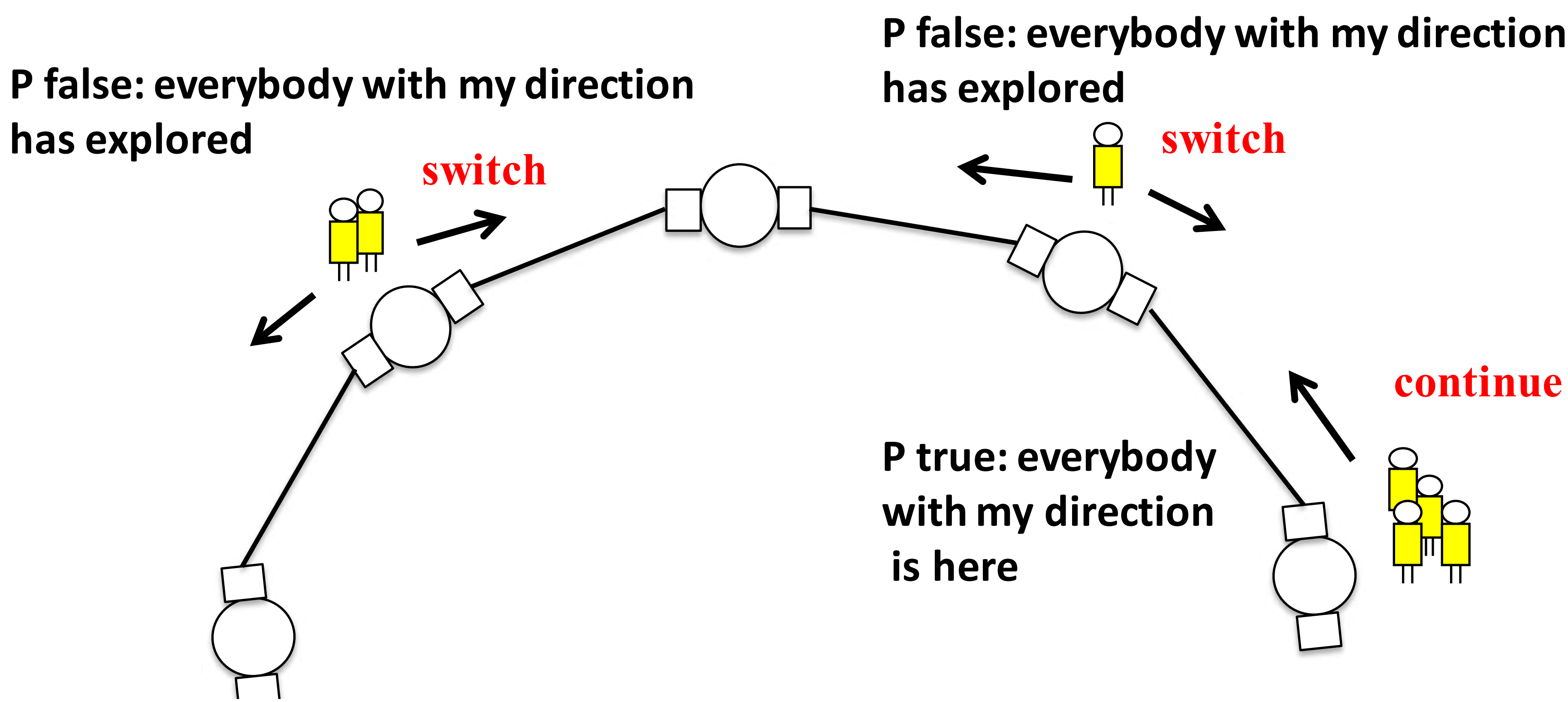
Move-left for 6n rounds



With Cross Detection: Without Chirality

Phase 1: Exploration

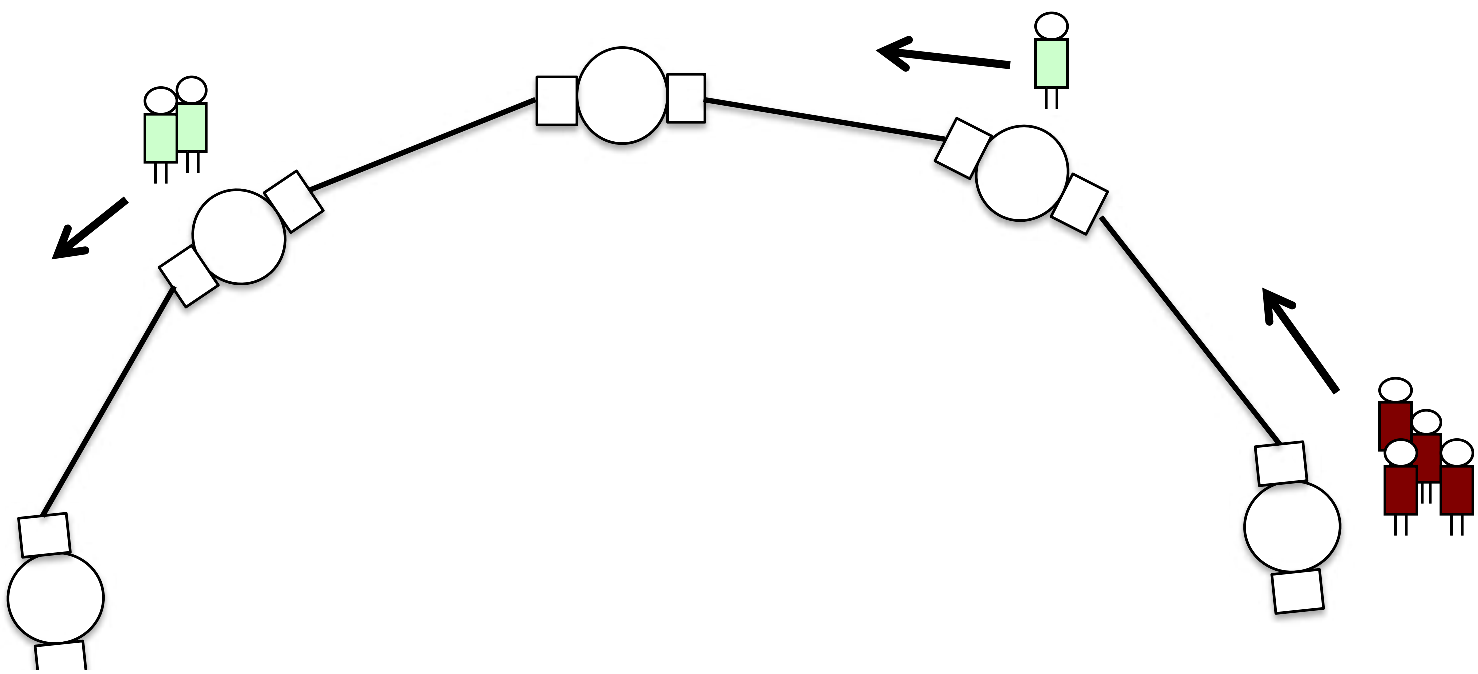
P: last time I met someone new going in my direction was less than 3**n** rounds ago; since then I traversed less than **n** links.



With Cross Detection: Without Chirality

Phase 1: Exploration

P: last time I met someone new going in my direction was less than 3**n** rounds ago; since then I traversed less than **n** links.

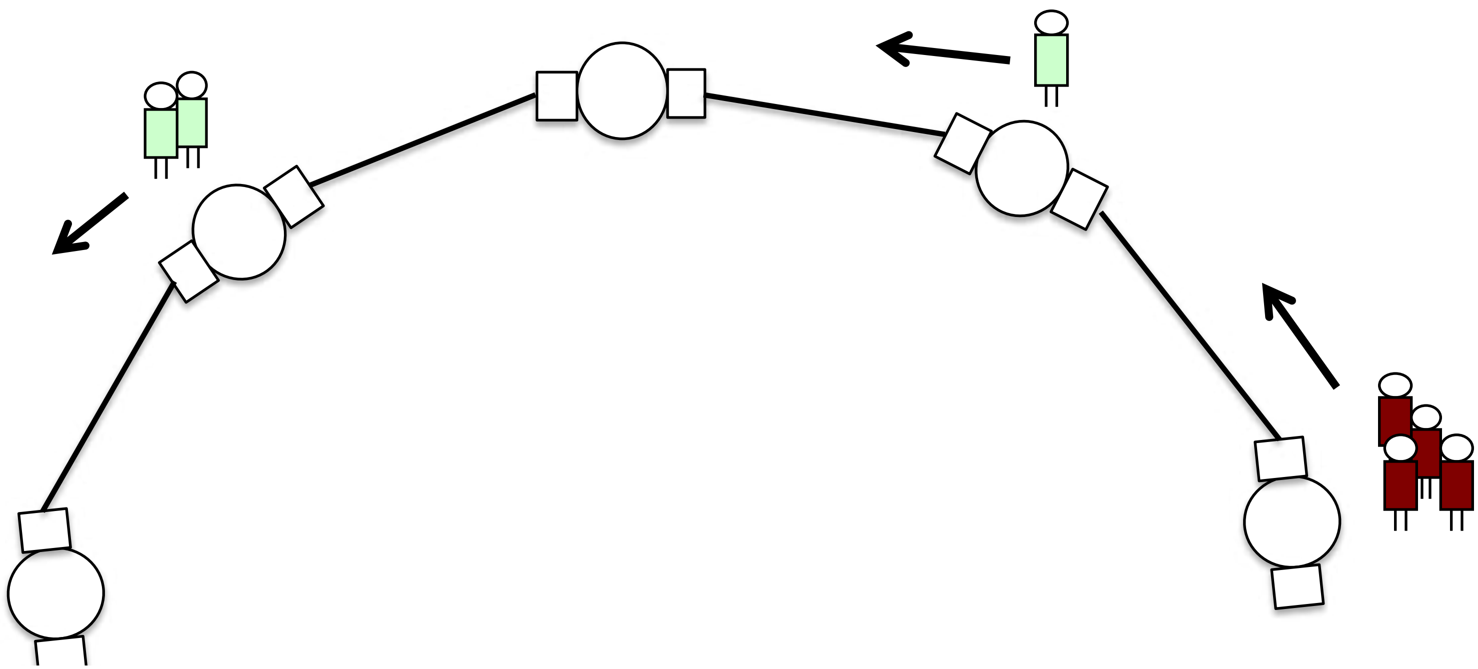


With Cross Detection: Without Chirality

Phase 1: Exploration

Switch Keep-moving in the new direction for *6n* rounds

Continue Keep-moving left for *6n* rounds

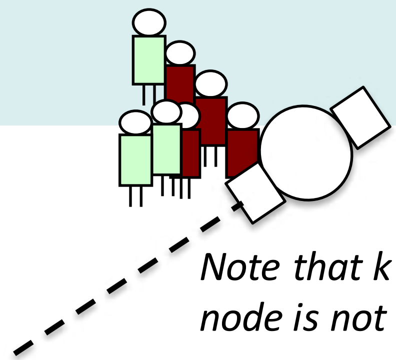


With Cross Detection: Without Chirality

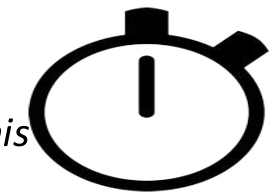
Phase 1: Exploration

Switch

At round 12n: if there are *k* agents in this node AND crossed less than *n* links AND met someone less than *9n* rounds ago AND never met anybody in opposite direction: **TERMINATE**
Otherwise: **Phase 2**



Note that k agents in this node is not sufficient



Round 12 n

Continue

At round 12n: if crossed less than *n* links and met someone less than *9n* rounds ago: **TERMINATE**
Otherwise: **Phase 2**

With Cross Detection: Without Chirality

Phase 1: Exploration

If an agent terminates in Phase 1, then all agents terminate and gathering has been correctly achieved. Otherwise, no agent terminates and all of them have done a complete tour of the ring.

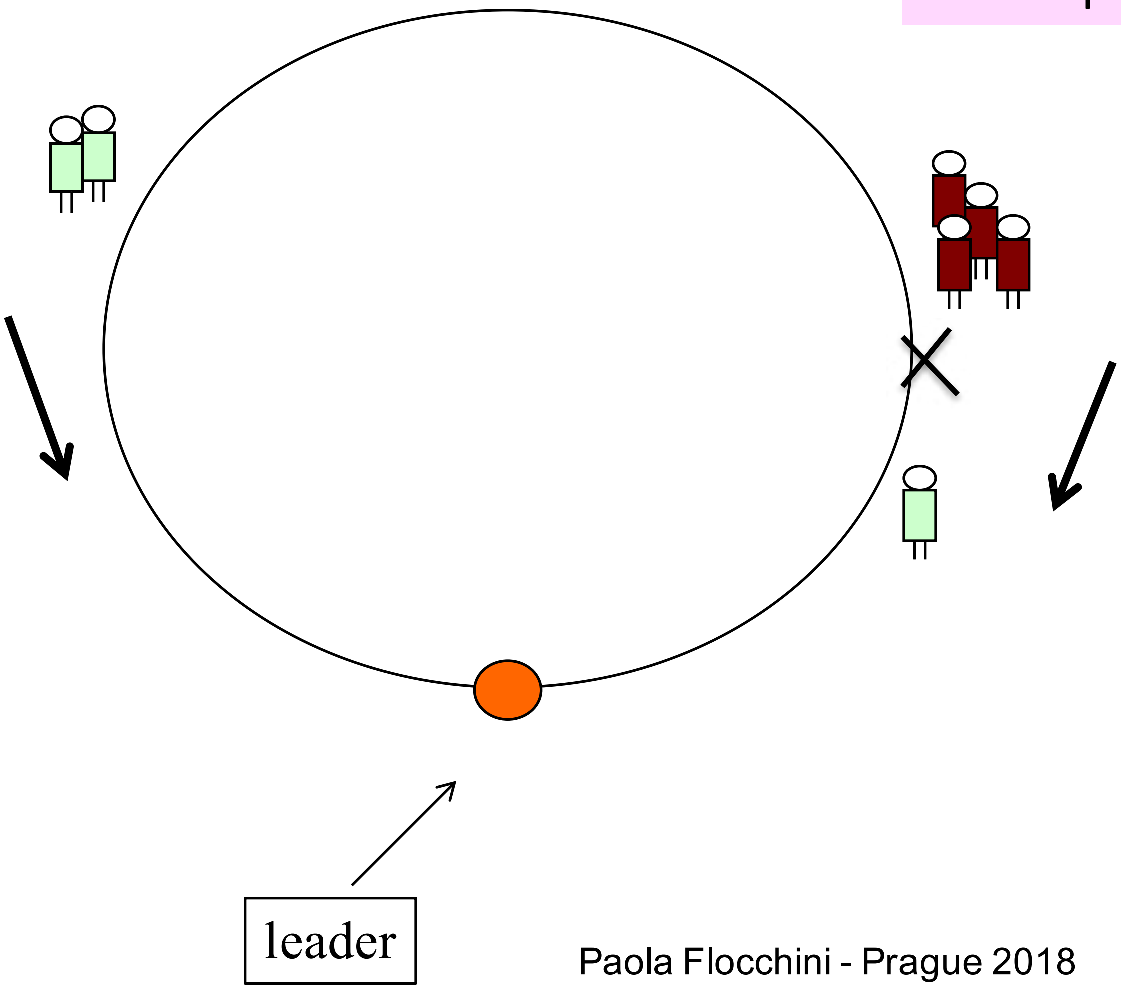
Phase 2: Gathering

The agents know the configuration and know if gathering is feasible. If it is, they all elect the same leader (edge or node) and they start the phase moving towards it through the shortest path.

Gathering in Dynamic Rings

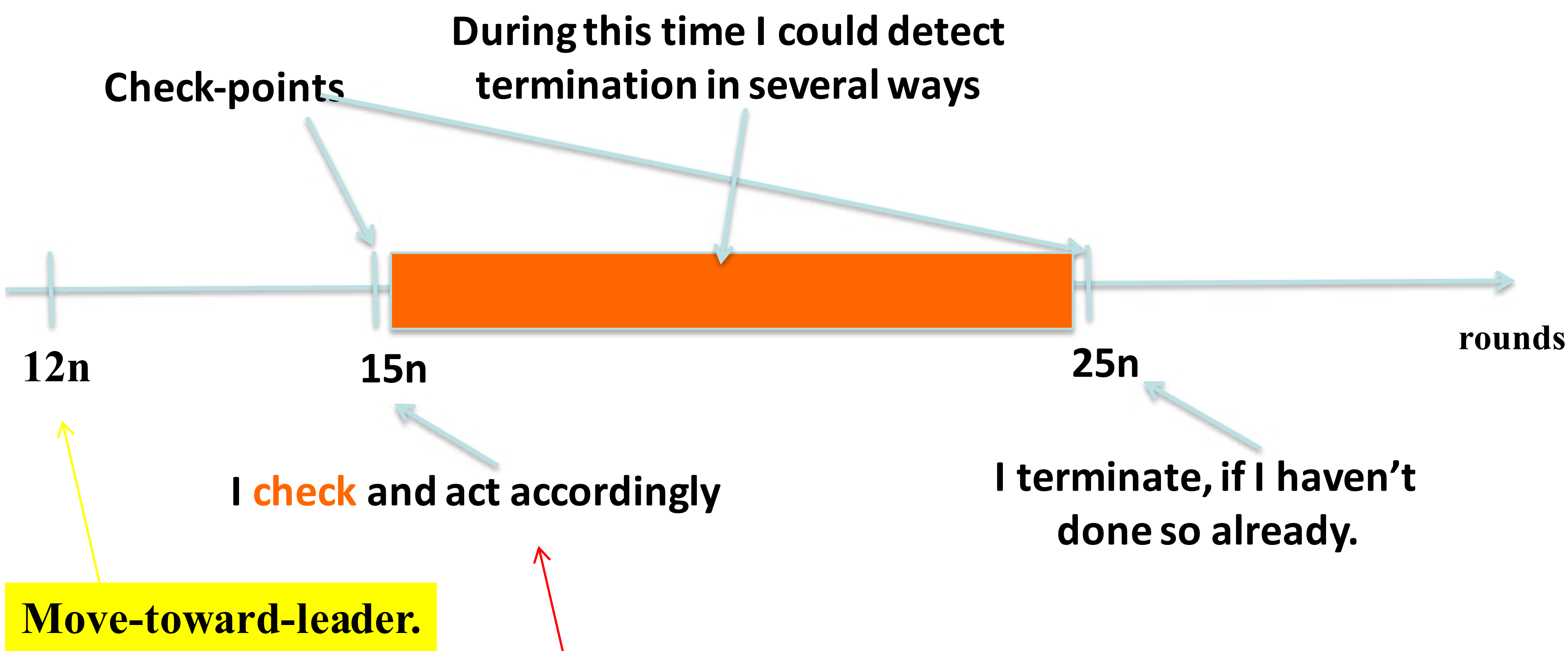
$\mathcal{A}$

Aperiodic configuration



With Cross Detection: Without Chirality

Phase 2: Gathering



If k of us are here TERMINATE

If I reached the leader, I become *ReachedElected* and switch direction

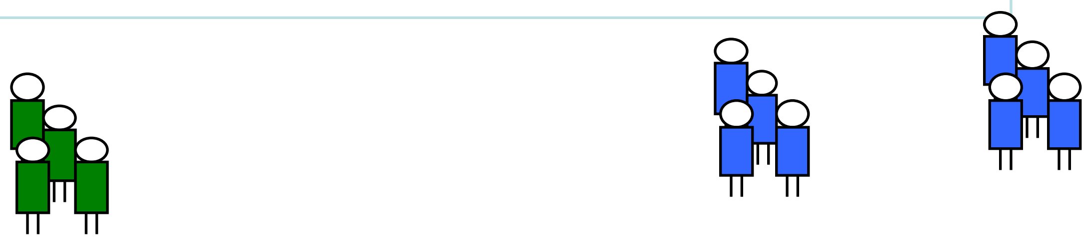
If I did not reach the leader I become *ReachingElected* and keep moving

With Cross Detection: Without Chirality

Phase 2: Gathering

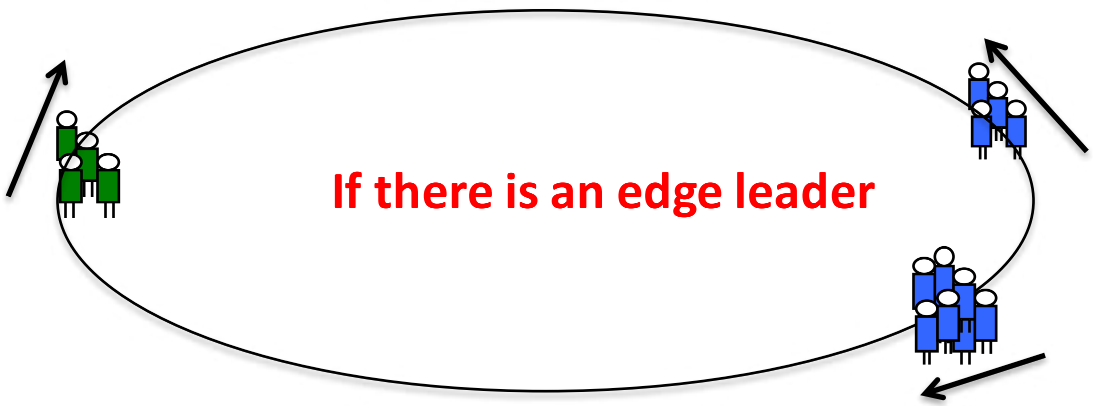
round 15n

At round 15n, there is at most one group in state ***ReachingElected***, and at most two groups in state ***ReachedElected***.



With Cross Detection: Without Chirality

Phase 2: Gathering

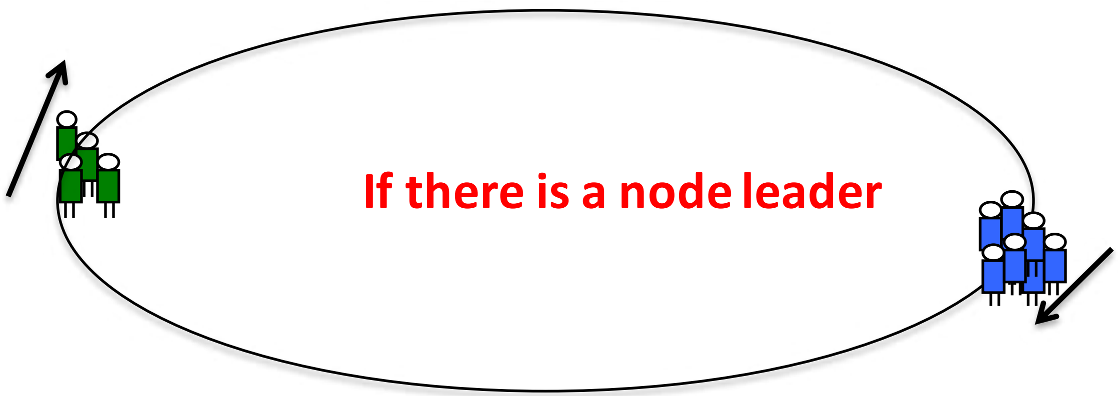


there are two groups of agents in state *ReachedElected* with opposite direction toward the *ReachingElected* *group*

With Cross Detection: Without Chirality

Phase 2: Gathering

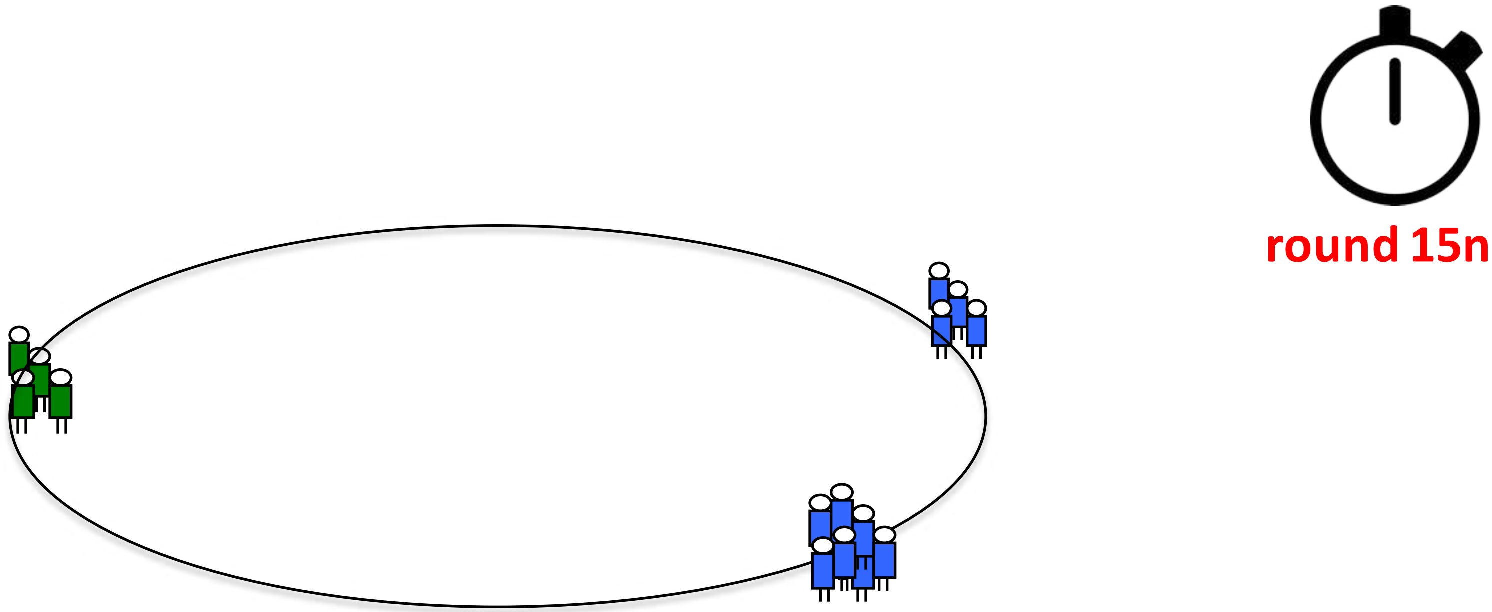
round 15n



there is a unique group of agents in state *ReachedElected*

With Cross Detection: Without Chirality

Phase 2: Gathering

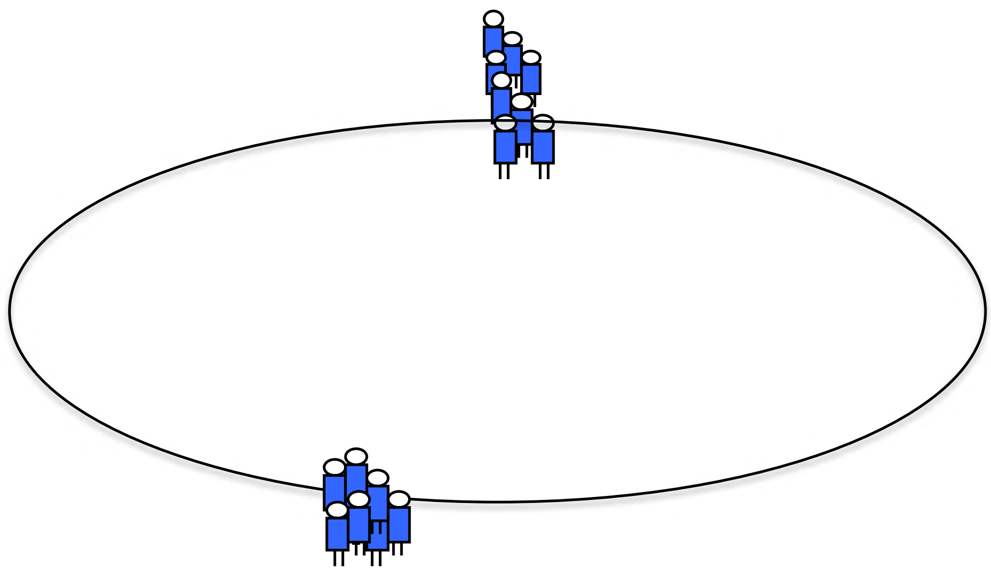


-the ***ReachingElected*** agents **switch direction** and try to reach the agents ***ReachedElected*** to join them

- the ***ReachedElected*** agents **keep same direction** and try to gather.

With Cross Detection: Without Chirality

Phase 2: Gathering



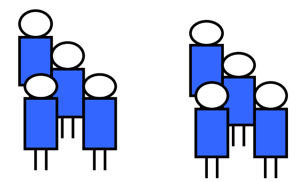
But the missing link can create several situations to be taken care of …

With Cross Detection: Without Chirality

Phase 2: Gathering

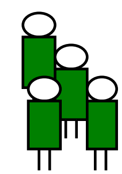
ReachedElected

If I cross a group of agents, I switch direction to try to catch them.
If they cross me again (double-crossing), TERMINATE
If they do not cross me a second time (i.e., I join them) switch direction again and stay in *ReachedElected* state



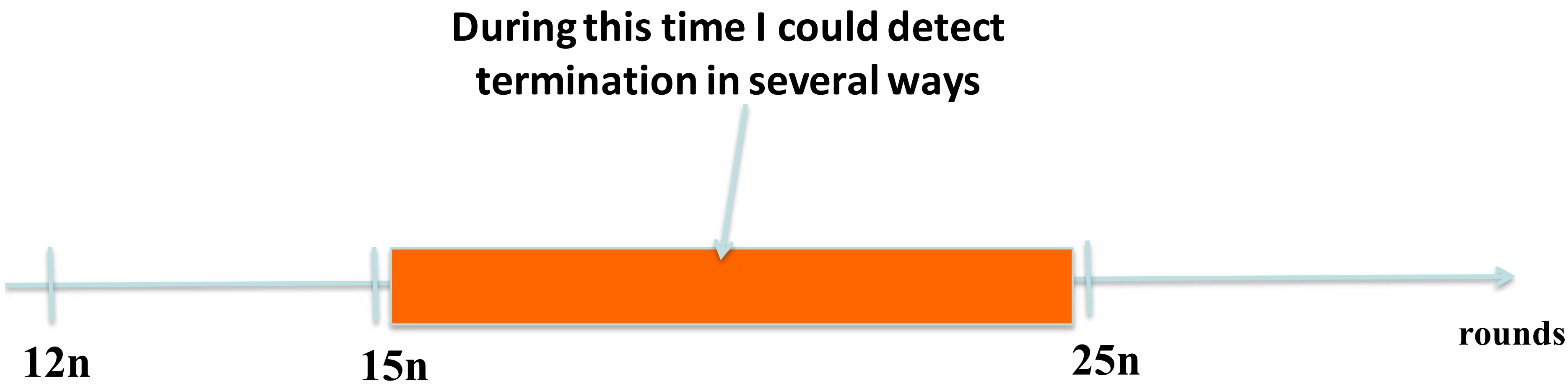
ReachingElected

If I reach the leader: switch direction and become *ReachedElected*
If I am blocked at a missing edge and I am reached by some other agent I become *ReachedElected* and I keep my direction
If I cross some other agent, I stop and wait.
if I meet anybody new while waiting in the next 2n rounds, switch direction and become *ReachedElected*; otherwise TERMINATE



With Cross Detection: Without Chirality

Phase 2: Gathering



TERMINATE

In state ***ReachedElected***

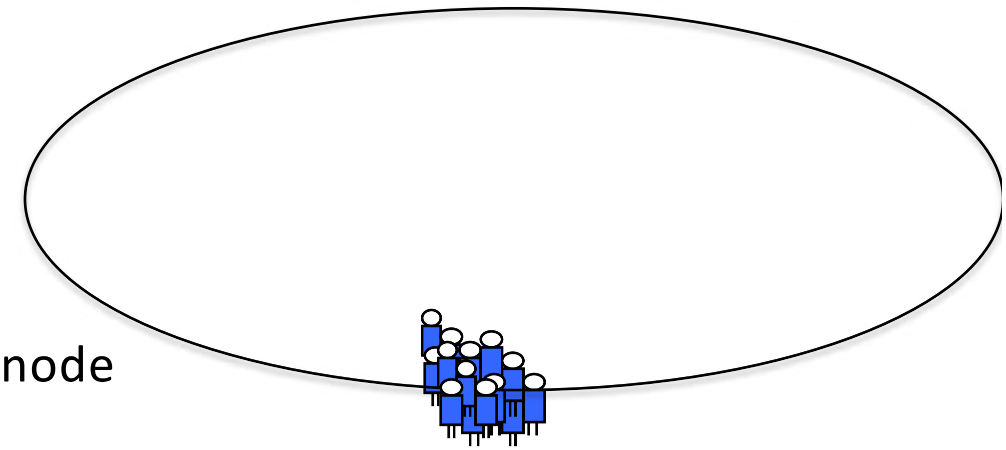
- double-crossing a group of agents

With Cross Detection: Without Chirality

Phase 2: Gathering

TERMINATE

In any state:

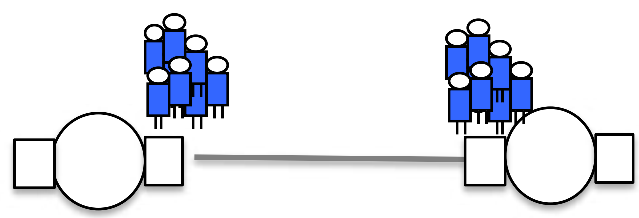


k **agents on same node**

Gathering is achieved on this node

blocked on a missing edge for *2n* rounds

If nobody reached us by now, the other group is on the other side of the edge and Gathering is achieved on this edge

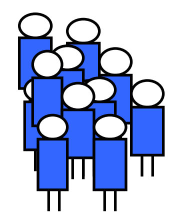


With Cross Detection: Without Chirality

Phase 2: Gathering

Round 25n

Phase 2 terminates correctly by round 25n.



GATHERING: COSTS

TIME	chirality	no chirality
cross detection	$\mathcal{A}$	$\mathcal{A}$ O(n)
no cross detection	$\mathcal{A}$	$\mathcal{A} \setminus \mathcal{E}$

With knowledge of n

GATHERING: COSTS

TIME	chirality	no chirality
cross detection	$\mathcal{A}$ O(n)	$\mathcal{A}$ O(n)
no cross detection	$\mathcal{A}$ O(n log n)	$\mathcal{A} \setminus \mathcal{E}$ $O(n^2)$

With knowledge of n

Mobile Agents in Time-Varying Graphs

EXPLORATION

Paola Flocchini - Prague 2018

Mobile Agents in Time-Varying Graphs

EXPLORATION

- C. Avin, M. Koucký, Z. Lotker. How to explore a fast-changing world (cover time of a simple random walk on evolving graphs). (*ICALP* 2008).
- D. Ilcinkas, A.M.Wade. On the Power of Waiting when Exploring Public Transportation Systems. (*OPODIS* 2011)
- P. Flocchini, M. Kellett, P.C. Mason, N. Santoro. Searching for Black Holes in Subways. *Theory of Computing Systems*, 2012.
- P. Flocchini, B. Mans, N. Santoro. On the exploration of time-varying networks. *Theoretical Computer Science,* 2013.
- D. Ilcinkas, A.M.Wade Exploration of the *T* -Interval-Connected Dynamic Graphs: The Case of the Ring. (*SIROCCO* 2013).
- P. Flocchini, M. Kellett, P.C. Mason, N. Santoro. Mapping an unfriendly subway system. (*FUN* 2014)

Mobile Agents in Time-Varying Graphs

EXPLORATION

- D. Ilcinkas, R. Klasing, A.M.Wade. Exploration of constantly connected dynamic graphs based on cactuses. (*SIROCCO* 2014).
- E. Aaron, D. Krizanc, E. Meyerson. DMVP: Foremost waypoint coverage of Time-Varying Graphs, (*WG* 2014).
- T. Erlebach, M. Hoffmann, F. Kammer On Temporal Graph Exploration. (*ICALP 2015*)
- G.A. Di Luna, S. Dobrev, P. Flocchini, N. Santoro Exploring 1-interval-connected rings. (*ICDCS 2016*)
- M. Bournat, S. Dubois, and F. Petit, Computability of perpetual exploration in highly dynamic rings (*ICDCS 2017*)
- M. Bournat, A.K. Datta, and S. Dubois, Self-stabilizing robots in highly dynamic Environments (*SSS 2018*)

Time-Varying Graph

EXPLORATION

G.A. Di Luna, S. Dobrev, P. Flocchini, N. Santoro.
Exploring 1-interval-connected rings. (*ICDCS 2016*).

Termination

Explicit Termination

all agents terminate knowing that the ring has been explored.

Partial Termination

at least one agent terminates knowing that the ring has been explored.

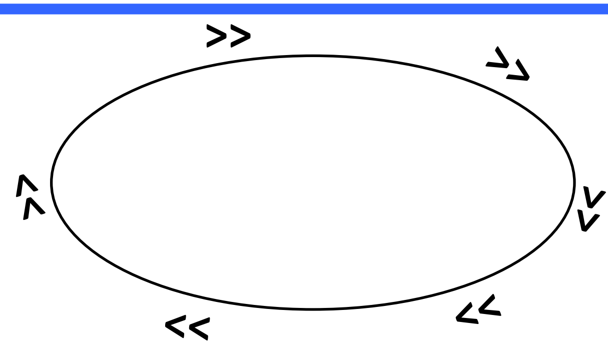
Main Questions:

Under what conditions is it possible to explore the dynamic ring ?

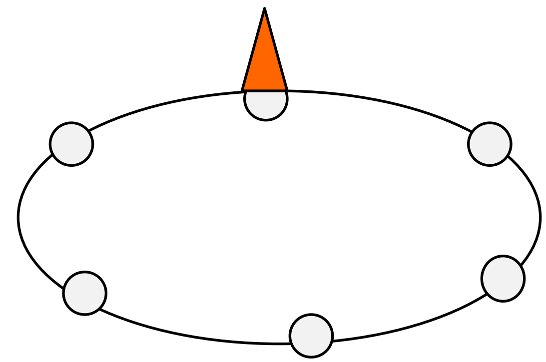
When can the agents explicitly terminate ?

What is the minimum number of agents necessary to explore ?

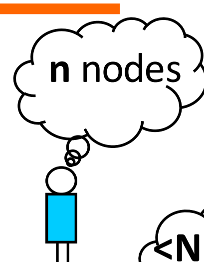
Important factors influencing feasibility/ termination



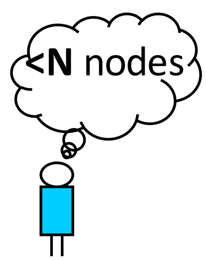
Chirality



Anonymity vs. presence of Landmark



Knowledge of exact size



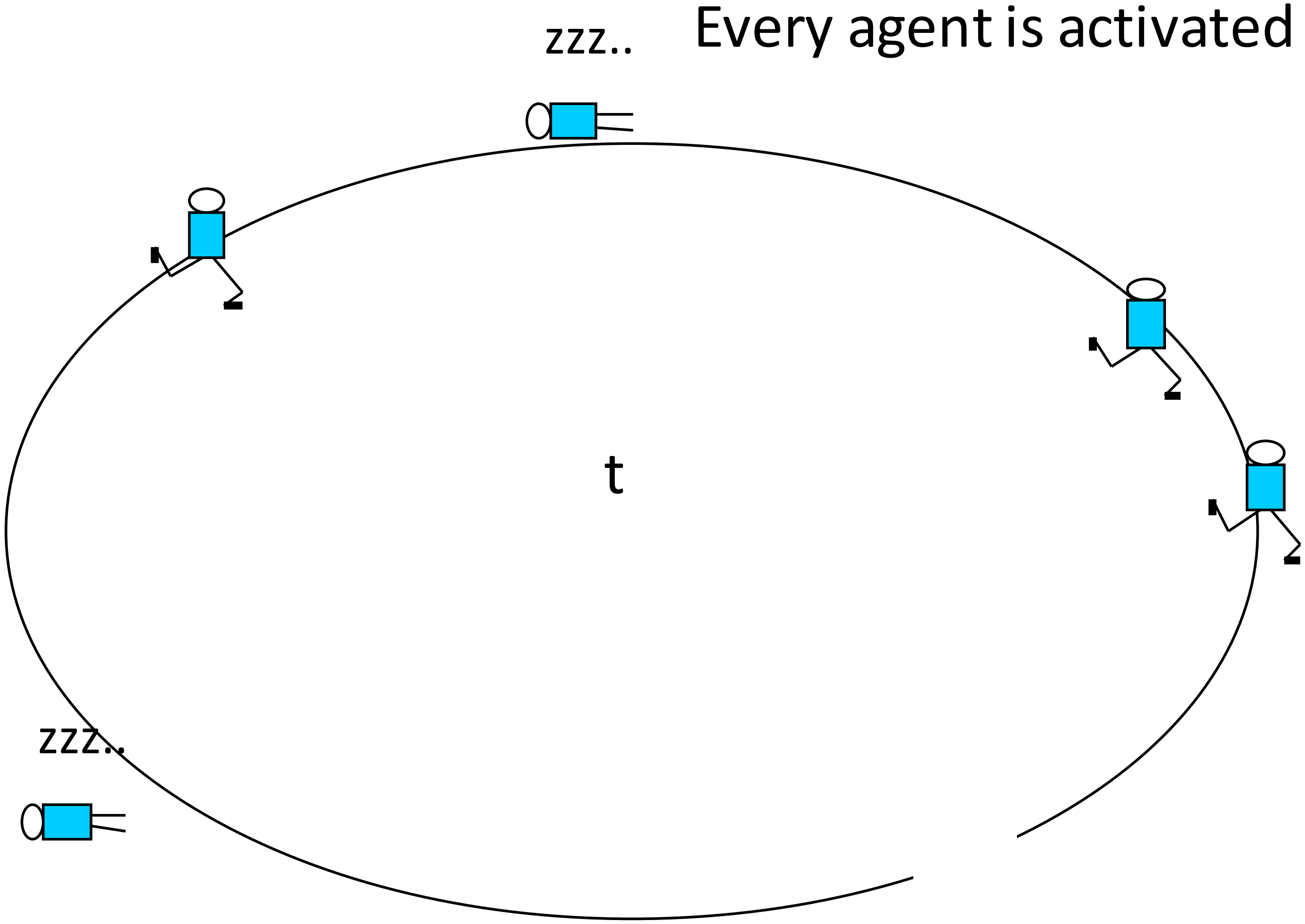
Knowledge of bound on size

Level of synchronicity

Semi-Synchronous (SSYNC)

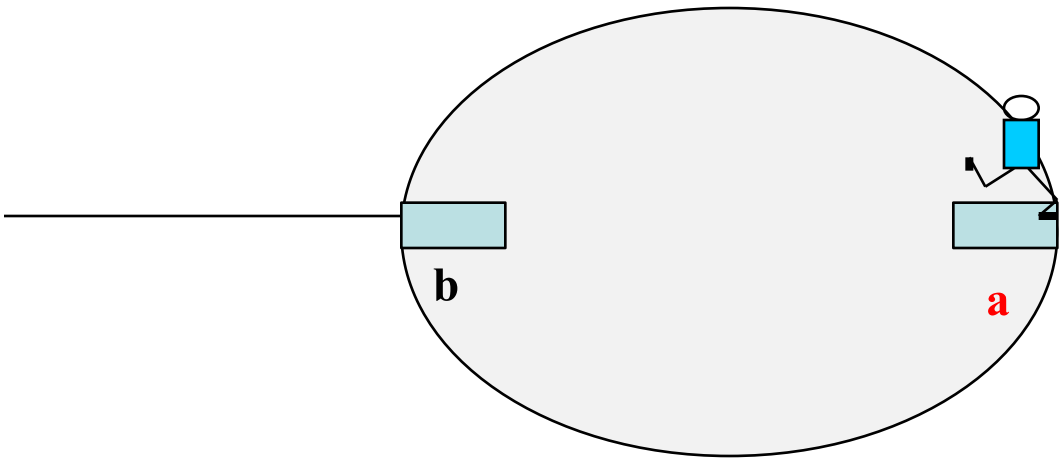
Not all agents are activated at every round

Every agent is activated infinitely often



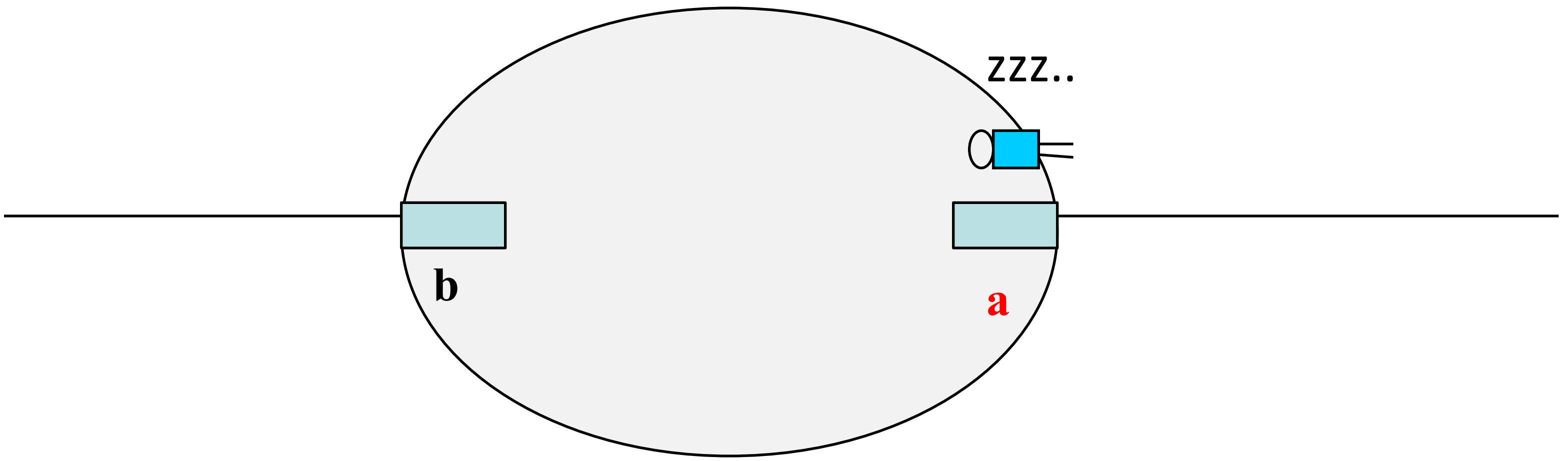
SSYNC

Not all agents are activated at every round



SSYNC

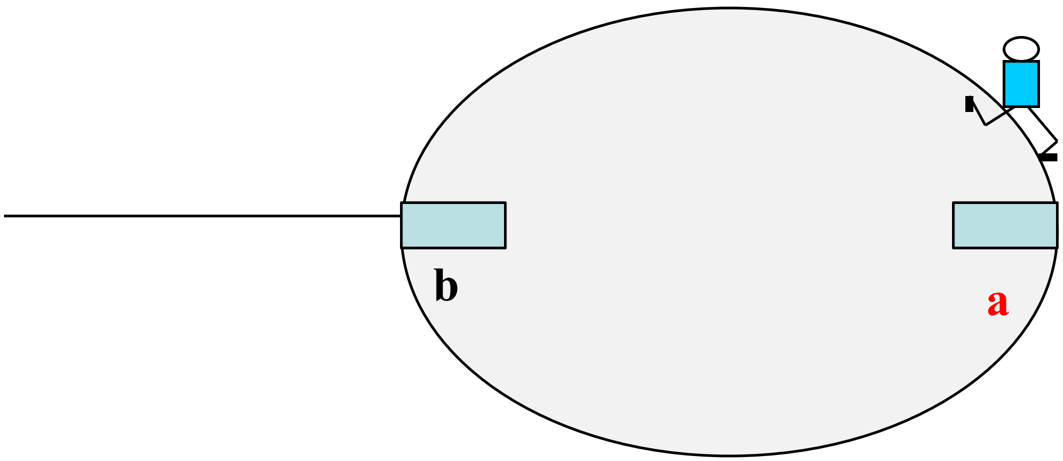
Not all agents are activated at every round



The agent might be sleeping next time the link appears ….

SSYNC

Not all agents are activated at every round

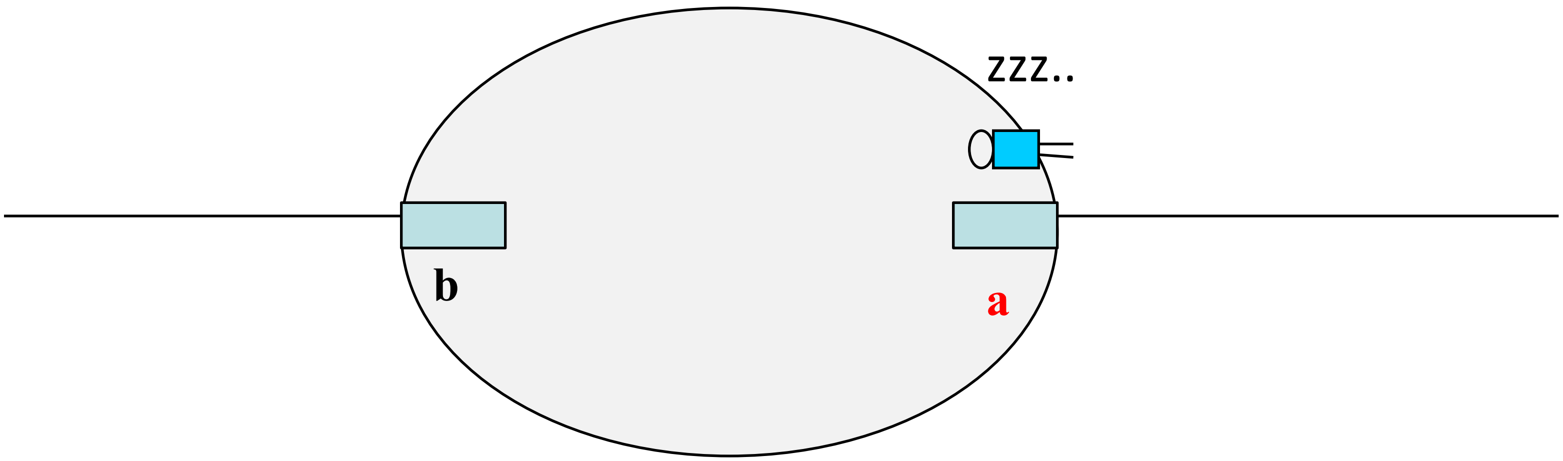


The agent might be sleeping next time the link appears ….

The link may be missing next time the agent is active …

SSYNC

Not all agents are activated at every round

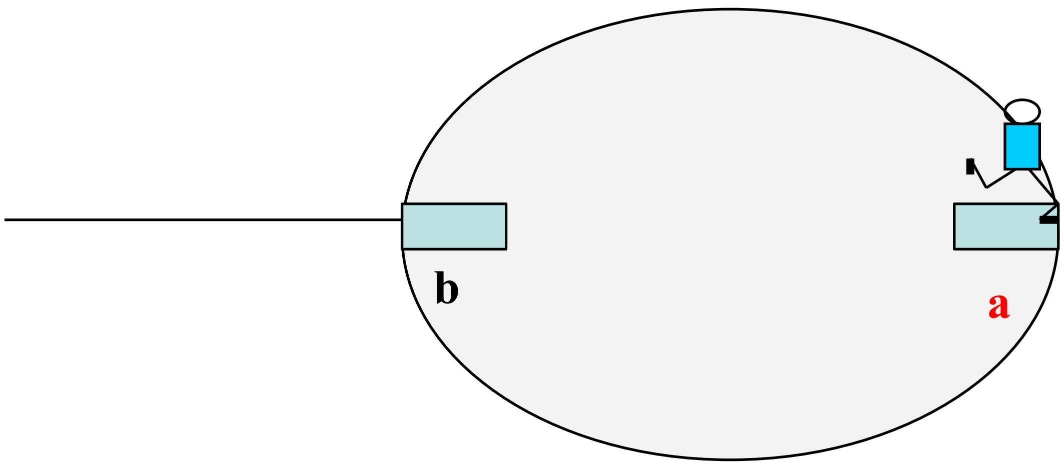


The agent might be sleeping next time the link appears ….

The link may be missing next time the agent is active …

The agent may be sleeping every time it appears !!!

SSYNC



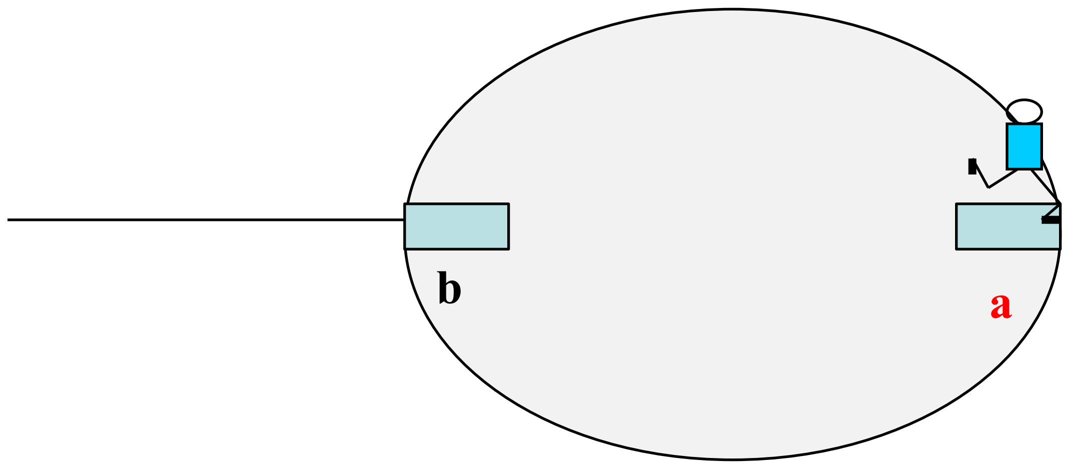
When activated, an agent finds itself on a port with a missing link

NS - No Simultaneity: can move only when active and link is present

ET- Eventual Transport: the agent will be eventually active at a time when the link is present

PT- Passive Transport: as soon as the edge is present the agent moves (even if not active).

SSYNC

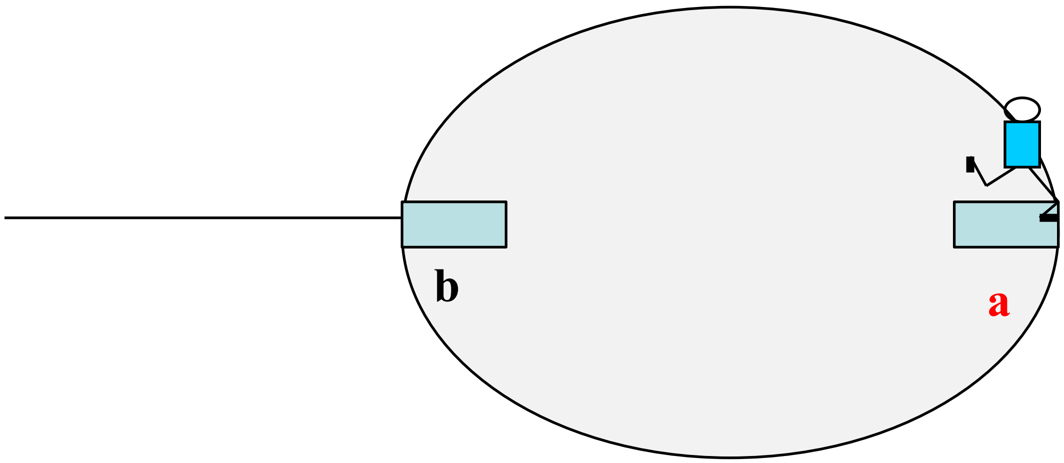


NS - No Simultaneity: can move only when active and link is present

The agent may be sleeping every time it appears !!!

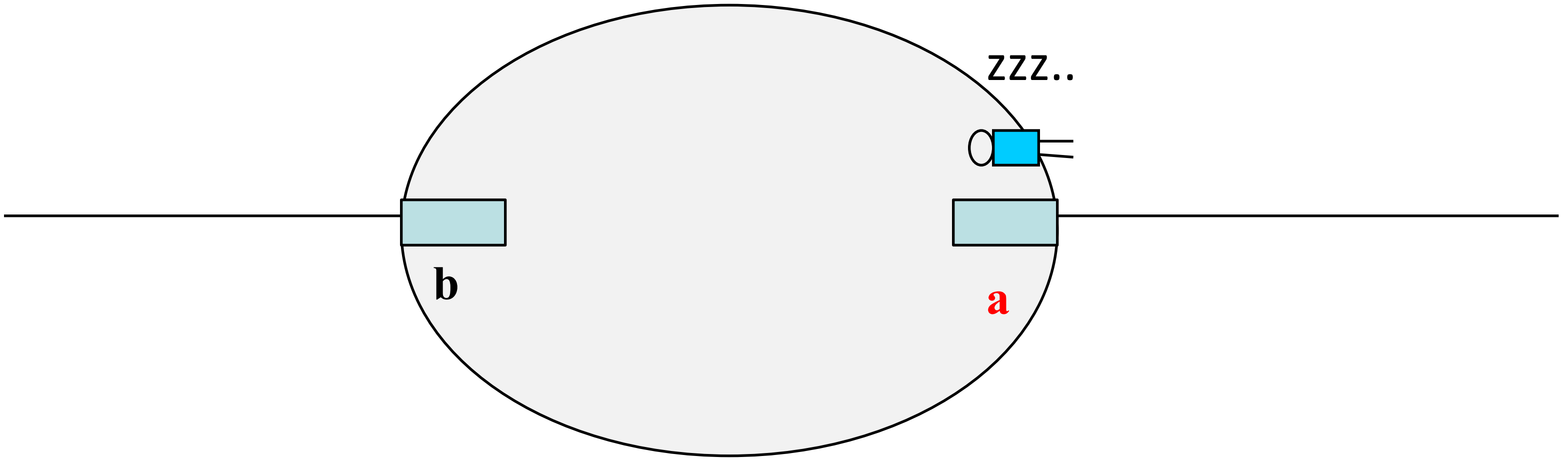
In NS exploration with any number of agents is impossible (even if if there is chirality, Knowledge of n, and a landmark)

SSYNC



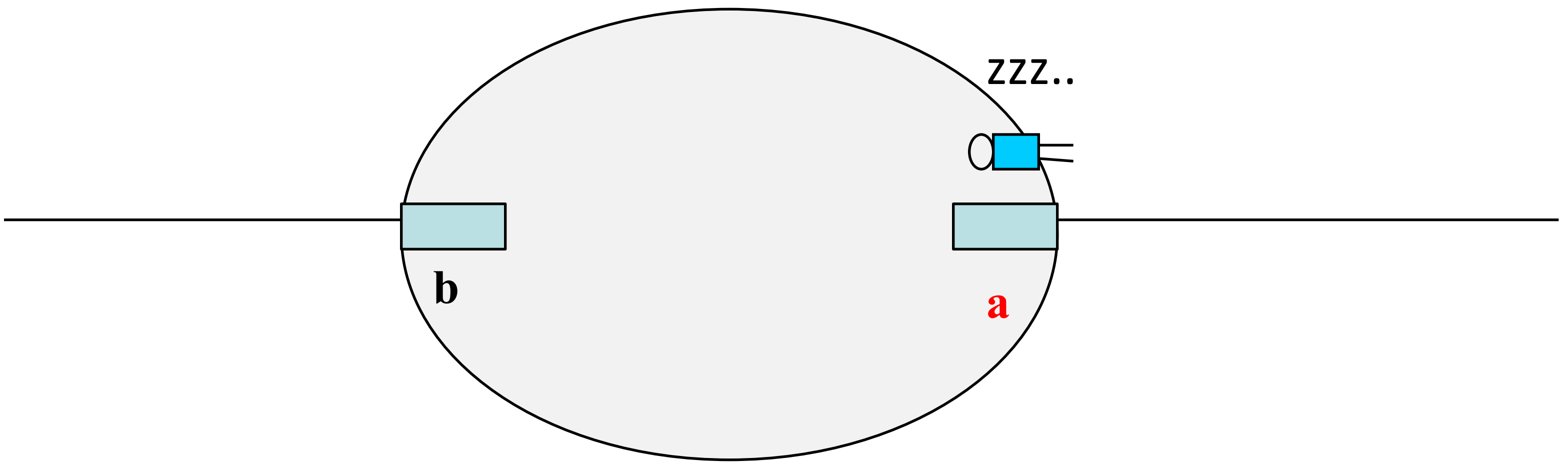
ET- Eventual Transport: the agent will be eventually active at a time when the link is present

SSYNC



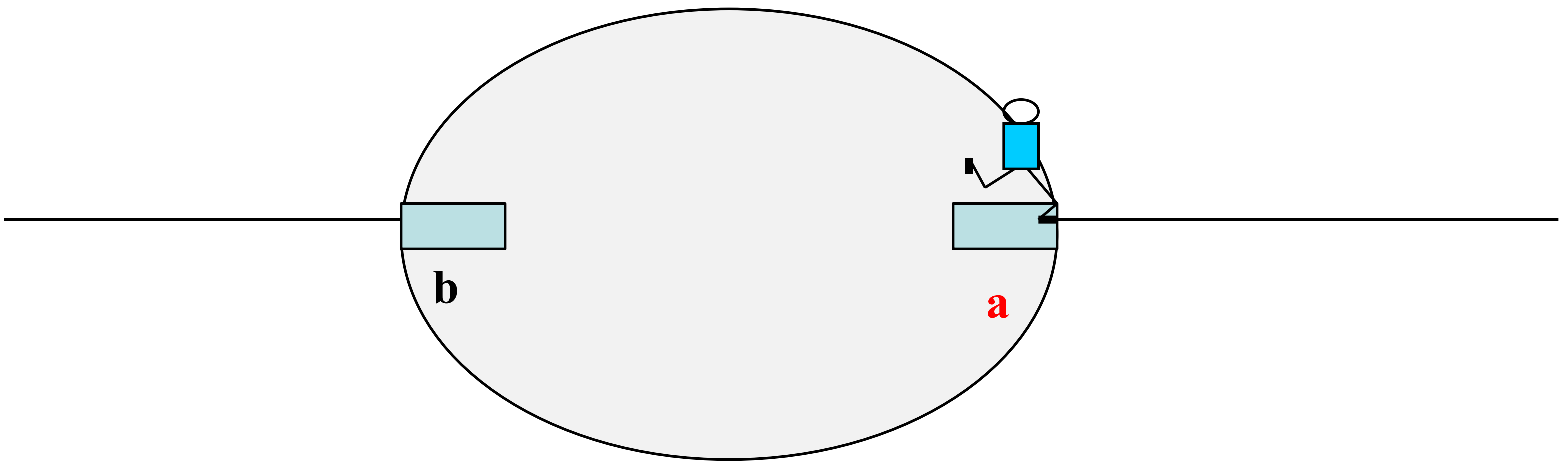
ET- Eventual Transport: the agent will be eventually active at a time when the link is present

SSYNC



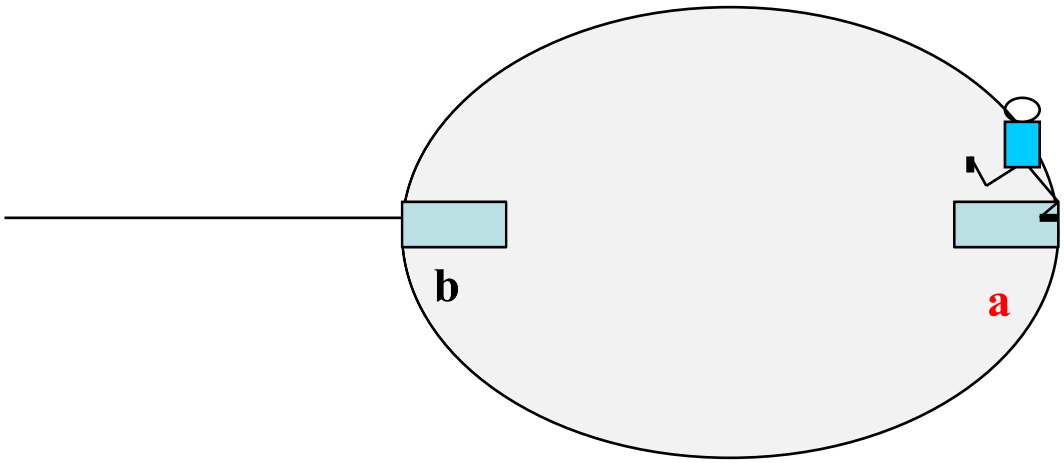
ET- Eventual Transport: the agent will be eventually active at a time when the link is present

SSYNC



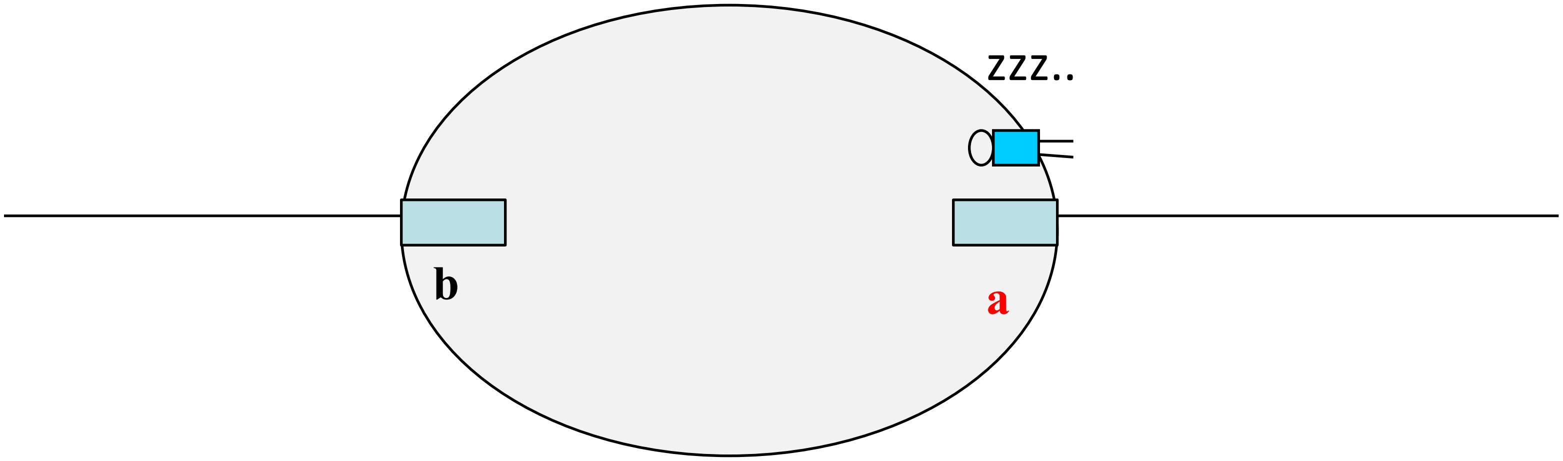
ET- Eventual Transport: the agent will be eventually active at a time when the link is present

SSYNC



PT- Passive Transport: as soon as the edge is present the agent moves (even if not active).

SSYNC



PT- Passive Transport: as soon as the edge is present the agent moves (even if not active).

SSYNC – Passive Transport (PT) - Impossibilities

Explicit Termination of 2 agents is impossible (even with chirality, knowledge of n and a landmark)

Without chirality, exploration with 2 agents is impossible (even if n is known and there is a landmark)

SSYNC – Passive Transport (PT) - Impossibilities

Explicit Termination of 2 agents is impossible (even with chirality, knowledge of n and a landmark)

Without chirality, exploration with 2 agents is impossible (even if n is known and there is a landmark)

Note that, even without dynamics:

Without an Upper Bound and without landmark, exploration with 2 agents is impossible (even if there is chirality)

SSYNC – Passive Transport (PT) – Possibility results

2	Chirality Known Bound N	$O(N^2)$ Partial termination
2	Chirality & Landmark	$O(n^2)$ Partial termination

SSYNC – Passive Transport (PT) – Possibility results

2	Chirality Known Bound N	$O(N^2)$ Partial termination
2	Chirality & Landmark	$O(n^2)$ Partial termination

Necessary

Necessary without Landmark

$\Omega(n\,N)$

Explicit Termination impossible

$\Omega(n^2)$

SSYNC – Passive Transport (PT) – Possibility results

Necessary

Necessary without Landmark

2	Chirality Known Bound N	$O(N^2)$ Partial termination
2	Chirality & Landmark	$O(n^2)$ Partial termination
3	Known Bound N	$O(N^2)$ Partial termination
3	Landmark	$O(n^2)$ Partial termination

Explicit Termination impossible

SSYNC – with Chirality and Known Upper Bound N

	Assumptions	Complexity
2 agents **SSYNC- PT** **anynomous**	**Upper Bound N** **Chirality**	**O(N^2)** Ω(n N) is a Lower Bound

Partial termination

Without an Upper Bound, exploration with 2 agents of an anonymous ring is impossible (even if there is chirality)

even without dynamics

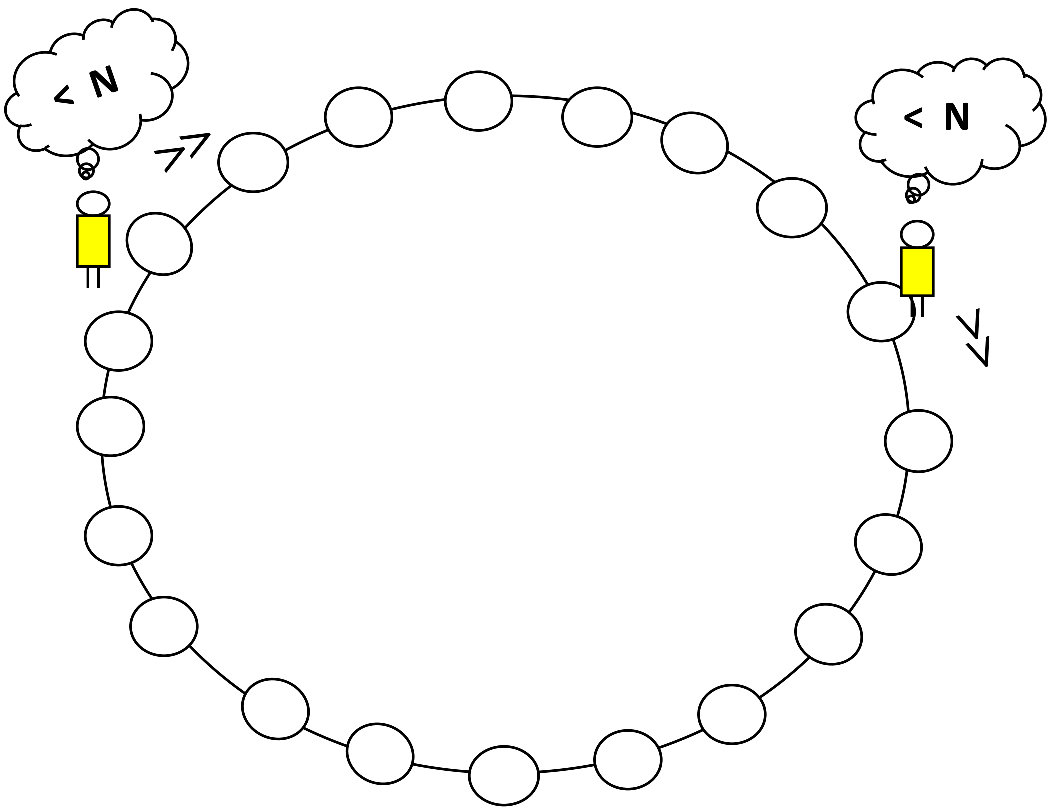
Without chirality, exploration with 2 agents is impossible (even with an Upper Bound)

because of dynamics

Explicit Termination is impossible

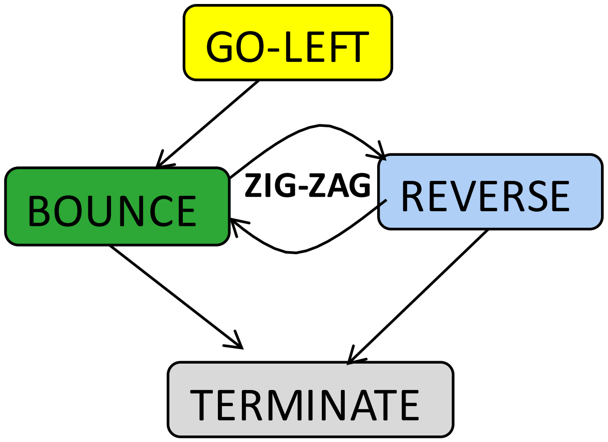
SSYNC – with Chirality and Known Upper Bound N

2 PT	Chirality Upper Bound N	$O(N^2)$ Partial termination



SSYNC – with Chirality and Known Upper Bound N

ZIG-ZAG



Moving left: either in state INIT or in state REVERSE

Moving right: always in state BOUNCE

SSYNC – with Chirality and Known Upper Bound N

ZIG-ZAG

BOUNCE when catching the other agent waiting at a missing link

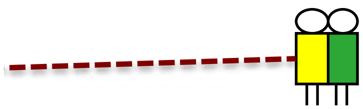
Left-to-right direction

SSYNC – with Chirality and Known Upper Bound N

ZIG-ZAG

BOUNCE when catching the other agent waiting at a missing link

Left-to-right direction



SSYNC – with Chirality and Known Upper Bound N

ZIG-ZAG

BOUNCE when catching the other agent waiting at a missing link

Left-to-right direction

REVERSE when finding an empty missing link

Right-to-left direction

SSYNC – with Chirality and Known Upper Bound N

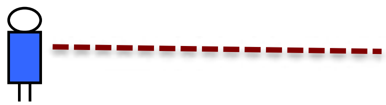
ZIG-ZAG

BOUNCE when catching the other agent waiting at a missing link

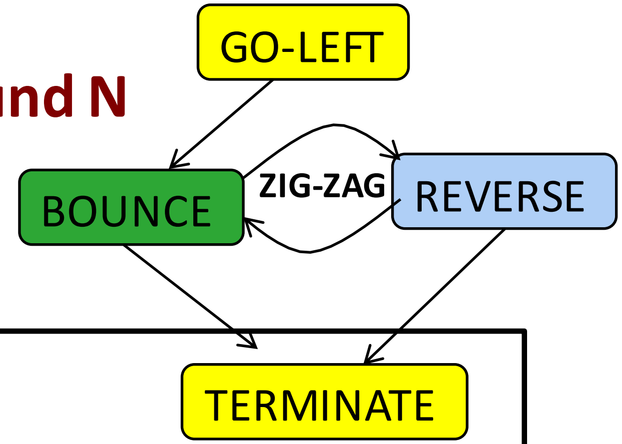
Left-to-right direction

REVERSE when finding an empty missing link

Right-to-left direction



SSYNC – with Chirality and Known Upper Bound N



ZIG-ZAG

GO-LEFT

If find a blocked edge with the other agent waiting in the left port, become BOUNCE, switch direction and starts moving right.

If, in state BOUNCE, find a missing edge before having traversed N edges, switch direction and become REVERSE and continue

TERMINATION CONDITIONS

1) Discovering to have traversed N consecutive edges in the same direction:

2) Catching the other agent at a distance smaller than the one of the previous catch

SSYNC – with Chirality and Known Upper Bound N

ZIG-ZAG

A REVERSE (or Init) agent Bounces when it catches the other agent moving left

A BOUNCE agent Reverses when it finds a missing link moving right

The distance traveled left by an agent to catch the other agent **keeps increasing**

Except when the ring has been already explored, in which case **it may decrease**

An agent terminates in two ways:

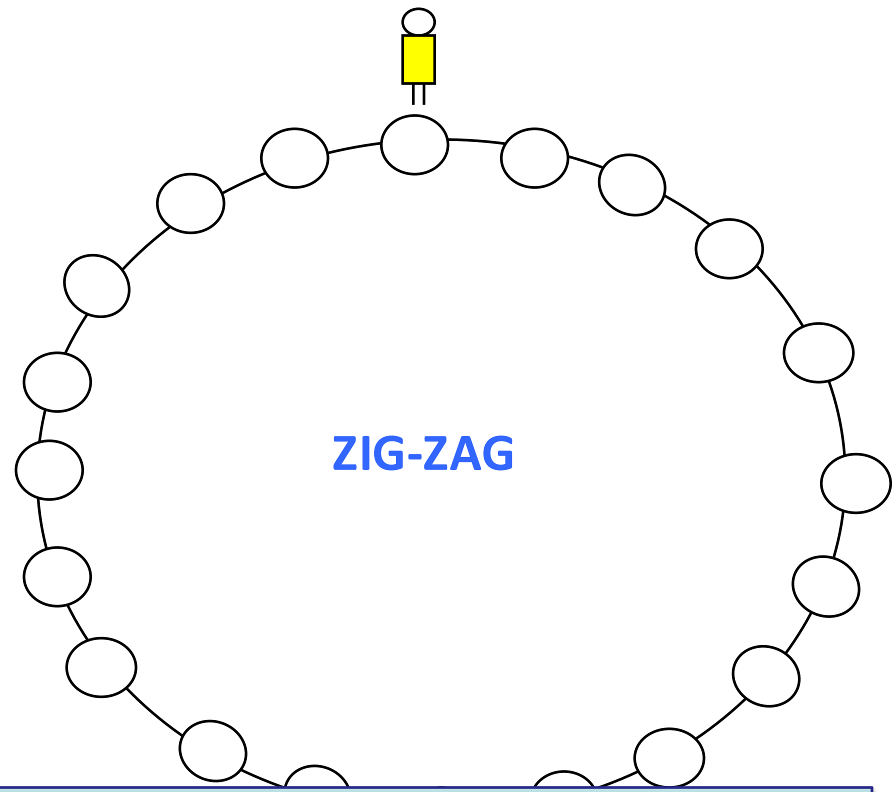
1) after vising N nodes (either in BOUNCE or REVERSE mode)
2) when noticing **such a decrease**

SSYNC – with Chirality and Known Upper Bound N

Number of moves: **$O(N^2)$**

$\Omega(n\ N)$ is
a Lower Bound

Partial Termination



Theorem
In SSYNC, with chirality and knowledge of an upper bound on the ring size, the ring can be explored with partial termination in $O(N^2)$ rounds.

SSYNC – with Chirality and Known Upper Bound N

	Assumptions	Complexity
2 agents **SSYNC- PT** **anynomous**	**Upper Bound N** **Chirality**	**$O(N^2)$** **Partial termination**

$\Omega(n\ N)$ is
a Lower Bound

Without an Upper Bound, exploration with 2 agents of an anonymous ring is impossible (even if there is chirality)

Even without dynamics

Without chirality, exploration with 2 agents is impossible (even with an Upper Bound)

Because of dynamics

Explicit Termination is impossible

SSYNC – with Chirality and Known Upper Bound N

	Assumptions	Complexity
2 agents SSYNC- PT ~~anynomous~~	~~Upper Bound N~~ Landmark Chirality	~~$O(N^2)$~~ $O(n^2)$ $\Omega(n^2)$ is a Lower Bound

Partial termination

Without an Upper Bound, exploration with 2 agents of an anonymous ring is impossible (even if there is chirality)

Even without dynamics

Without chirality, exploration with 2 agents is impossible (even with an Upper Bound)

Because of dynamics

Explicit Termination is impossible

SSYNC – Possibility results

	Agents	Assumptions	Result
PT	2	Chirality Known Bound N	$O(N^2)$ Partial termination
	2	Chirality & Landmark	$O(n^2)$ Partial termination
	3	Known Bound N	$O(N^2)$ Partial termination
	3	Landmark	$O(n^2)$ Partial termination
ET	2	Chirality	Unconscious exploration
	3	Known n	Finite number of moves Partial termination

SSYNC – Impossibility results

	Agents	Assumptions	Even if	Result
NS	Any	None	Chirality, Known n, Landmark, distinct Ids	Impossible
PT	2	No chirality, anonymous	Known n, Landmark	Impossible
	2	None	Chirality, known n, Landmark	Explicit termination impossible
ET	Any	Landmark	Known bound N, Chirality, Landmark, Distinct Ids	Partial termination impossible

FSYNC

Agents	Assumptions	Even if	Result
2	Size unknown No landmark	Non-anonymous Chirality	Termination Impossible
Any	Size unknown No landmark Anonymous	Chirality	Termination Impossible

Agents	Assumptions	Complexity
2	Known Bound N	3N-6 Explicit termination
2	Chirality and Landmark	O(n) Explicit termination
2	Landmark	O(n log n) Explicit termination

OPEN PROBLEMS Gathering

	chirality	no chirality
Improve the time bounds **without cross detection**	$O(n \log n)$	$O(n^2)$

Gathering in **other dynamic graphs**

Gathering with **different dynamics**

OPEN PROBLEMS Exploration

Small gaps between **upper** and **lower bounds**

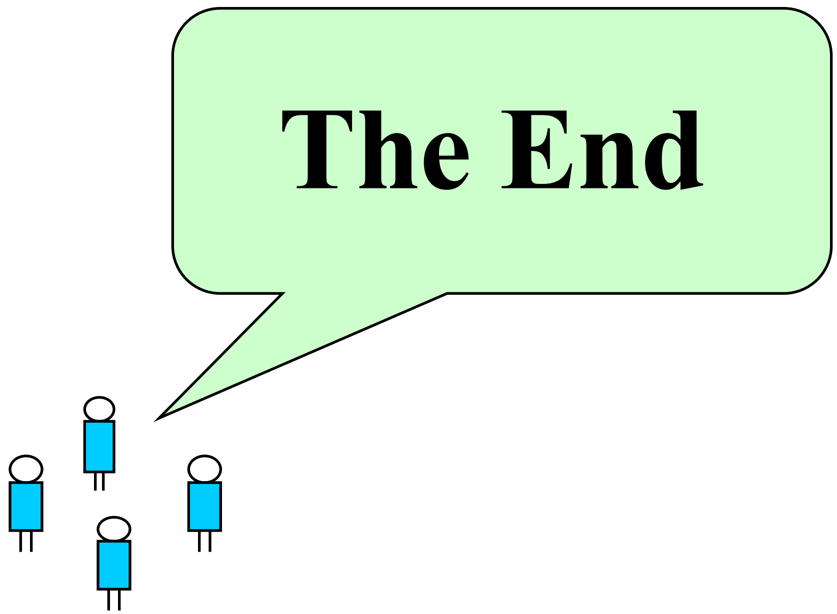
Exploration of **other dynamic graphs**

Exploration with **different dynamics**

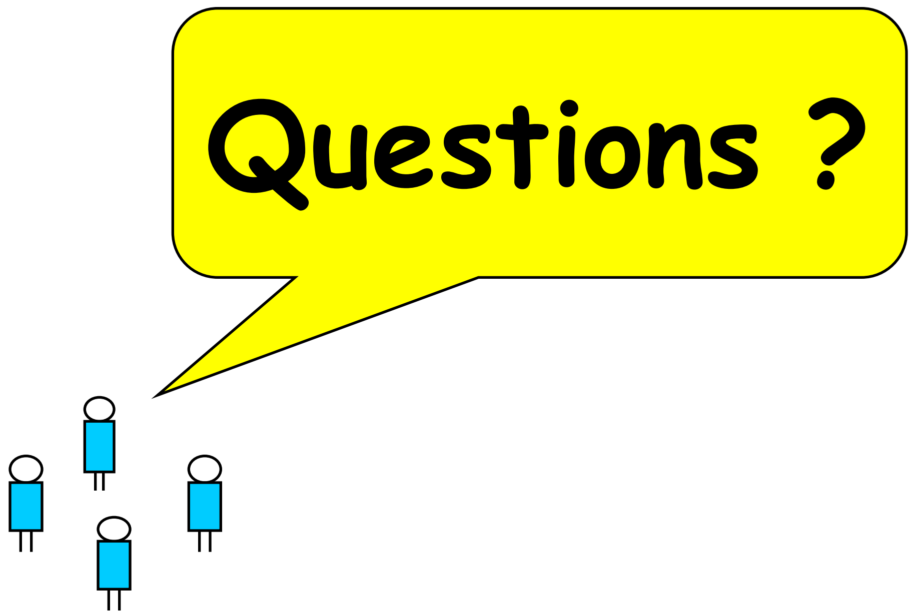
GENERAL CONCLUDING OBSERVATIONS

VERY LITTLE IS KNOWN
There is still a lot to discover

$c_{10} \longrightarrow c_9 \longrightarrow c_{11} \rightarrow c_{12}$ c_2


The End


Thank you


Questions ?


Thank you